

PAYING ATTENTION TO

# MATHEMATICS Education

# K-12

## Seven Foundational Principles for Improvement in Mathematics, K-12

- ❖ Focus on mathematics.
- ❖ Coordinate and strengthen mathematics leadership.
- ❖ Build understanding of effective mathematics instruction.
- ❖ Support collaborative professional learning in mathematics.
- ❖ Design a responsive mathematics learning environment.
- ❖ Provide assessment and evaluation in mathematics that supports student learning.
- ❖ Facilitate access to mathematics learning resources.

*“... every student should be equipped with the mathematical knowledge, skills and habits of mind that are essential for successful and rewarding participation in society.”*

– The Mathematics Working Group, Ontario Ministry of Education

# Paying Attention to Mathematics

Based on comparative assessments of student achievement on international tests, Ontario was ranked in 2007 as one of the top ten high-performing school systems in the world. A follow-up study in 2010 showed that Ontario along with four other jurisdictions (Singapore, Hong Kong, South Korea and Saxony, Germany) was able to sustain the gains it had made, leading to its ranking as a “great system.”<sup>1</sup> As a result of precisely targeted supports, resources and programming, Ontario students have enjoyed growing success. Ontario educators continue to work together to ensure that this success extends across all subject areas, K–12.

Yet, within the province, an essential area stands out as needing more attention.

Feedback from the field, including consultations and professional learning sessions, and from initial reviews of Board Improvement Plans for Student Achievement (BIPSAs), indicated a need for a closer look at and alignment of K–12 mathematics. All were in agreement that although gains were being made across the province in many areas, the time had come to pay focused collective attention to mathematics teaching, leading and learning.

## The Mathematics Working Group

With so much at stake for students who do not do well in mathematics, the Ministry of Education’s Student Achievement Division brought together a Mathematics Teaching and Learning Working Group in September 2010 to identify what it would take to bring greater depth and coherence to the K–12 mathematics program. Representing experts in mathematics pedagogy, curriculum and research, and including members from the ministry, district school boards, federations, professional associations and universities, the group adopted a highly collaborative approach in order to surface current, research-based knowledge in mathematics teaching and learning, K–12. Following research and consultation, the group’s task was to advise the Assistant Deputy Minister on ways to bring about a positive change in the field and to ensure better results for all students.

This publication shares seven foundational principles from the work to date of the Mathematics Working Group. These principles will serve as a guide for planning and implementing improvements in mathematics teaching and learning.

---

<sup>1</sup> Barber, M., & Mourshed, M. (2007). *How the world’s best-performing school systems come out on top*. McKinsey & Company. [www.mckinsey.com](http://www.mckinsey.com).

Mourshed, M., Chijioke, C., & Barber, M. (2010). *How the world’s most improved school systems keep getting better*. McKinsey and Company. [www.mckinsey.com](http://www.mckinsey.com)

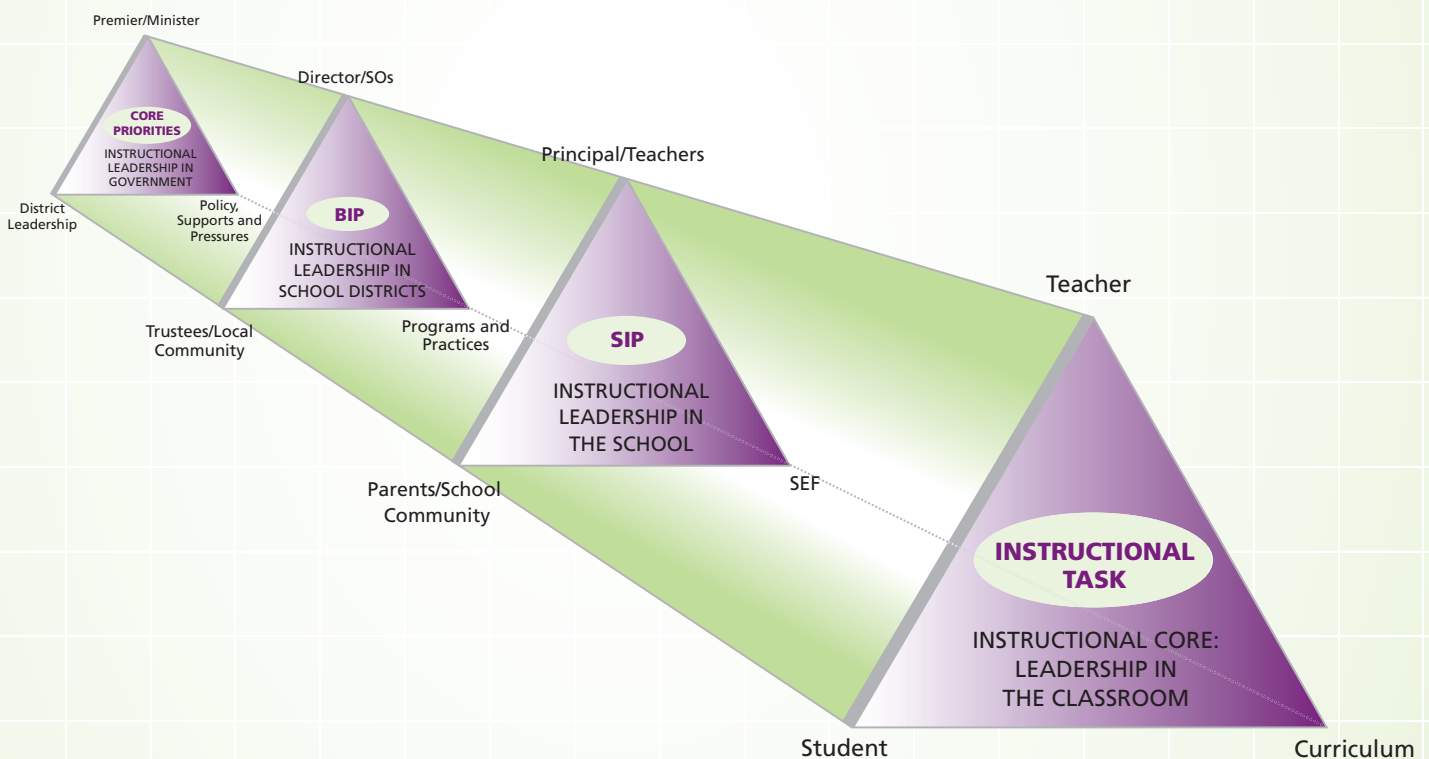
“Alignment of focus and energy on the instructional core is key to teaching, learning and leading in Ontario.”

– School Effectiveness Framework  
<http://resources.curriculum.org/secretariat/framework/index.shtml>

## An Invitation to Focus on the Instructional Core

Understanding that district school boards have well-established planning processes and are at different stages of implementing their own mathematics strategy, the Mathematics Working Group developed the foundational principles as a way to support – not to supplant – existing planning processes. They are intended both to spark conversation and to guide planning, helping educators at every level to identify the precise strategies and activities that will improve mathematics teaching and learning in the classroom. While each principle has a different focus, each addresses engaging all students in a thorough understanding of mathematics by supporting and strengthening the work in the classroom – referred to in the School Effectiveness Framework as “the instructional core.” With educators at all four levels of the education system working together to improve mathematics instruction and the mathematical experience of all students – from pre-school and the primary grades to the end of high school – improved learning and achievement will inevitably result.

*The “Prism,” familiar to Ontario educators in the School Effectiveness Framework, illustrates how provincial direction, district priorities and school improvement efforts can strengthen the instructional core in mathematics.*



## ❖ Foundational Principle

# Focus on mathematics.

*“As teachers become more familiar with which ideas are more complex for students and why, they are better able to ensure that their instruction is at the appropriate developmental level for students, and that it challenges students’ mathematical conceptions in appropriate ways. This minimizes the likelihood of students developing mathematical misconceptions.”*

– Small, M. (2009). *Making math meaningful to Canadian students, K–8*. Toronto, ON: Nelson Education.

All educators – classroom teachers, early childhood educators, school principals and other instructional leaders in the education system – use a variety of critical thinking and problem-solving strategies to engage all students in making connections between content and process as they work toward a thorough understanding of mathematics.

### *Focusing on mathematics involves ...*

- starting with the curriculum – for both content and instructional strategies
- planning, sequencing and connecting mathematical key concepts across the grades
- helping students explore and make sense of patterns and relationships between and among the strands
- enabling students to apply their mathematical knowledge and extend their understanding
- enabling students to develop a deep understanding of mathematical concepts, skills and processes
- engaging students as co-learners in the development, refinement and expression of mathematics, including multiple representations of mathematical concepts
- undertaking complex mathematical tasks that are meaningful to students and engage them in learning relevant and rigorous mathematics
- using the range of higher-level thinking skills in mathematical processes
  - problem solving
  - reasoning and proving
  - communicating
  - representing
  - connecting
  - reflecting
  - selecting tools and computational strategies
- encouraging multiple approaches for learning and actively doing mathematics
- planning learning tasks and opportunities through differentiated instruction
- fostering questioning – teacher to student, student to student, student to teacher – so that all can see the value in mathematics learning
- providing students with opportunities to consolidate their understanding of the mathematical processes and concepts
- developing learning goals and success criteria in response to student thinking and reasoning

## ❖ Foundational Principle

# Coordinate and strengthen mathematics leadership.

*“[Principals are] never going to have a credible conversation with a teacher about practice unless [they’ve] actually been doing it, repeating it, and getting good at it.”*

– Elmore, R. (2010, Summer). Leading the instructional core: An Interview with Richard Elmore. *In Conversation*, 11(3).

All system, board and school leaders commit to providing the resources that support the most effective teaching and learning of mathematics for all students.

### *Mathematics leaders are ...*

- system/board/school leaders who set direction and develop buy-in with respect to the teaching and learning of mathematics
- classroom teachers who monitor the impact of their practice in order to improve it as evidenced by student work
- all educators who are committed to action grounded in research and identified student need, informed by all data available on student achievement and student need in mathematics

### *Effective leadership in mathematics involves ...*

- ensuring ongoing and differentiated professional learning in mathematics for educators across the system
- ensuring supports are in place to promote networking of school leaders at the board, school and classroom level
- learning what good mathematics looks like in order to reflect on good practice – by participating in co-learning, co-planning and co-teaching
- encouraging risk-taking and innovation by providing a safe environment for co-learning
- encouraging collaboration through joint planning, shared debriefing, observation of teaching and key conversations on how practice informs learning
- supporting parent involvement by helping them understand shifts in the teaching and learning of mathematics

### *Mathematics leadership planning requires ...*

- focusing board and school improvement plans, based on classroom, school and system level student achievement data, on mathematics
- aligning resources, including staffing, with mathematics priorities to support students with the greatest mathematics need
- engaging in open-to-learning conversations regarding alternative ways to address student needs in mathematics
- building on professional learning to support teachers in their ongoing effective teaching and learning practices in mathematics
- participating in professional learning to develop knowledge of what good teaching and learning in mathematics looks like

# Build understanding of effective mathematics instruction.

*Five teaching practices for improving the quality of discourse in mathematics classrooms:*

*(1) “talk moves” that engage students/facilitate discussion, (2) the art of questioning, (3) using student thinking to propel discussions, (4) setting up supportive environments and (5) Orchestrating the discourse*

– Chapin, S. H., O’Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K–6 (2nd ed.)*. Sausalito, CA: Math Solutions.

All those who work in education understand that effective instruction in mathematics is essential because it equips students with knowledge, skills and habits of mind that are required for successful and rewarding participation in 21<sup>st</sup> century technological society.

## *Effective mathematics instruction is ...*

- based on problem solving and investigation of mathematical concepts
- built on students’ prior mathematical knowledge and understanding, and relevant to students’ lives
- differentiated to meet the diversity of students’ learning needs
- based on mathematical and pedagogical knowledge and understanding of students’ mathematical development
- grounded in the belief that all knowledge has to be constructed by each student in a learning environment in which mathematical knowledge-building and understanding is nurtured

## *Effective mathematics instruction involves ...*

- engaging students in all of the mathematical processes
- honouring multiple ways of mathematical thinking, reasoning, meaning-making, connection-making and developing concepts
- providing a variety of materials to help students represent problem-situations with a variety of representations and tools
- developing a mathematics learning community with opportunities for student-student as well as teacher-student talk and interaction
- responding to students’ thinking by providing timely feedback with opportunities to act on that feedback and planning next steps for instruction
- focusing on key concepts in the mathematics curriculum
- ensuring cross-curricular connections
- developing authentic tasks founded on key concepts in mathematics
- providing assessment and evaluation in mathematics that is fair, transparent and equitable
- ensuring an intentional use of learning experiences and selecting technologies, manipulatives and content knowledge to involve students in learning actively

## *Planning effective mathematics instruction requires ...*

- professional learning opportunities for administrators and teachers that build knowledge of mathematics for teaching and learning
- classroom tasks that are practical and relevant to students’ lives to develop their mathematics understanding, problem-solving skills, and related technological skills that they can apply in their daily lives and in the future workplace
- community partnerships that foster the integration of parents as partners in supporting student achievement in mathematics

## ❖ Foundational Principle

# Support collaborative professional learning in mathematics.

*“We conceptualize teacher professional learning as embedded in the classroom context and constructed through experience and practice in sustained iterative cycles of goal setting/planning, practicing, and reflecting.”*

– Bruce, C. D., Esmonde, I., Ross J., Dookie, L., & Beatty, R. (2010). The effects of sustained classroom-embedded teacher professional learning on teacher efficacy and related student achievement. *Teaching and Teacher Education*, 26, 1598–1608.

All those who teach and inquire into mathematics and mathematics education are engaged as co-learners in professional learning inquiries, working collaboratively to promote the most effective teaching and learning of mathematics for all students.

### *Professional learning in mathematics is ...*

- informed by students’ mathematical thinking as evidenced by a range of data about ALL students
- built on inquiry where educators, facilitators and researchers engage as co-learners in exploring areas of mutual interest
- oriented to participants in a collaborative culture of curiosity and risk-taking
- based on high-quality research and instruction
- focused on mathematics knowledge for teaching, including content knowledge, pedagogy and pedagogical content knowledge
- differentiated by the educator’s readiness and needs and informed by students’ mathematics learning

### *Mathematics professional learning inquiries involve ...*

- a focus on students, student thinking and student demonstrations of understanding
- asset models of learning where all participants consider themselves learners of mathematics and of ways of teaching mathematics
- an integrated blend of learning inside and outside of classrooms with a commitment to collaborative inquiry
- an iterative, cyclical approach informed by monitoring and implementation, and knowledge dissemination
- long-range planning through sustained support, multi-year initiatives and implementation toward a gradual release of responsibility model for teachers
- classroom-embedded learning, online and remote access, and electronic learning opportunities that capitalize on the use of technology

### *Mathematics professional learning planning requires ...*

- articulation of explicit mathematics goals and strategies across schools and boards including clear statements in Board Improvement Plans for Student Achievement
- alignment of mathematics goals and strategies across system levels and grades
- development of mathematics leadership capacity including mathematical knowledge for teaching
- building board and system capacity to strengthen mathematics learning and teaching approaches in classroom practice aligned with current curriculum and ministry support documents

## ❖ Foundational Principle

# Design a responsive mathematics learning environment.

*“Effective teachers ... know that a wrong answer might indicate unexpected thinking rather than a lack of understanding; equally, a correct answer may be arrived at via faulty thinking.”*

– Anthony, G., & Walshaw, M. (2009). Effective pedagogy in mathematics. *International Bureau of Education, Educational Practice Series, 19*. Retrieved from: [http://www.ibe.unesco.org/fileadmin/user\\_upload/Publications/Educational\\_Practices/EdPractices\\_19.pdf](http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf).

The learning environment in all classrooms reflects the commitment of the teacher, the school and the board to meeting the needs of all students in the teaching and learning of mathematics.

### *A responsive mathematics learning environment involves ...*

- meeting the social-emotional needs of all students, K–12, by ensuring:
  - respect for their developmental needs
  - support for risk-taking in learning mathematics
  - positive attitudes and beliefs about mathematics
  - collaboratively constructed class norms
- optimizing the physical set-up of the classroom by ensuring:
  - space for collaborative work
  - equitable access to a variety of tools, mathematics learning resources, concrete materials, technology and manipulatives
  - the display of student thinking that reflects the mathematics topics currently being taught

### *Planning a responsive mathematics learning environment requires ...*

- knowledge and capacity in the intentional teaching of mathematics
- communication skills to support an inquiry approach to the teaching and learning of mathematics
- knowledge of classroom dynamics, including ways to:
  - support students in transitions
  - recognize the diversity of the classroom as an asset to learning
  - honour the range of mathematics knowledge students bring to the classroom
  - build student engagement and respect student voice
  - develop an open-to-learning stance among students



## ❖ Foundational Principle

# Provide assessment and evaluation in mathematics that supports student learning.

*“...teachers generally listen for the ‘correct’ answer instead of listening for what they can learn about the students’ thinking; they listen evaluatively rather than interpretively. The teachers with whom we have worked have tried to address this issue by asking students questions that either prompt students to think or provide teachers with information that they can use to adjust instruction to meet learning needs.”*

– Leahy, S., Lyon, C., Thompson, M., & William, D. (2005). *Classroom assessment: Minute by minute, day by day. Educational Leadership, 63* (3), 18–24.

All educators provide assessment and evaluation in mathematics that is fair, transparent and equitable to determine the mathematical literacy of all students and to support their ongoing mathematics learning.

### *Fair assessment and evaluation in mathematics involves ...*

- providing multiple opportunities for students to demonstrate the full range of their learning
- careful planning to relate curriculum expectations and learning goals to the interests, learning styles and preferences, needs and experiences of all students
- timing assessment of learning and evaluation after students have had multiple opportunities to demonstrate their learning and received descriptive feedback
- using varied and parallel instructional experiences, including manipulatives and technologies

### *Transparent assessment and evaluation in mathematics involves ...*

- clear communication to students and parents at the beginning of the school year/course and at other appropriate points throughout the school year/course
- ongoing descriptive feedback that is clear, specific, meaningful and timely to support improved learning and achievement
- sharing learning goals and co-constructing success criteria with students and linking them to mathematics that matters to them

### *Equitable assessment and evaluation in mathematics involves ...*

- supporting all students, with attention those with special education needs, those who are learning the language of instruction (English or French) and those who are First Nation, Métis or Inuit
- developing student self-assessment skills to enable them to assess their own learning, set specific goals and plan next steps for their learning
- focusing on the same knowledge and skills, while differentiating to meet student needs
- providing opportunities for students to demonstrate their growth on expected learning goals (e.g., modified/adjusted for readiness), and their achievement against the provincial standard (current reporting policy) so that teachers can:
  - provide feedback on student performance relative to learning goals
  - use this information to inform instruction

## ❖ Foundational Principle

# Facilitate access to mathematics learning resources.

*“Teachers have a critical role to play in ensuring that tools are used effectively to support students to organize their mathematical reasoning and support their sense-making.”*

– Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 36 (5), 412–446.

System, board and school leaders allocate mathematics learning resources, both human and material, on the basis of a needs assessment and evidence-based decision-making.

### *Mathematics learning resources are ...*

- an integrated part of the learning at all levels of the educational system
- built into system-wide strategic planning


### *Effective use of mathematics learning resources involves ...*

- appraising various criteria, including:
  - aligns with instructional practices identified as promising/effective by researchers and expert practitioners (based on both international and local knowledge/experience)
  - supports a learning need and adding value to the learning objective(s)
  - addresses improved student achievement
- increasing opportunities for teachers to collaborate in mathematics co-learning with the support of ministry, board and school administrators
- sharing “successful practices” learned from other schools, teachers and boards of the province, both within and beyond the classroom
- mobilizing mathematical teaching and learning knowledge among teachers, schools and boards across the province

### *Mathematics learning resource planning requires ...*

- allocation of mathematics learning resources aligned with board and school planning, goals and priorities and revisited on a cyclical basis
- a coherent and systematic approach to equitable access to resources across the system
- differentiation of resources according to need to facilitate different entry points for conceptual learning
- access to meaningful professional learning on how to use mathematics learning resources for pedagogical practices
- focus on the development of shared understanding of terms, concepts and constructs of mathematics
- development of a system to communicate availability of resources at all levels across the system
- provision of appropriate mathematics learning resources to support learning at all levels across the system





To order additional copies contact ServiceOntario

416-326-5300 or 1-800-668-9938

<http://www.publications.serviceontario.ca/ecom>



Printed on recycled paper

ISBN 978-1-4435-7305-4 Print

ISBN 978-1-4435-7306-1 PDF

© Queen's Printer for Ontario, 2011