Leaders in Mathematical Thinking
Connie Quadrini - Concrete and Digital Learning Tools

>> We've been very intentional about introducing tools, concrete tools and digital tools, not just because they're hands-on, although that is a good thing, but because those tools actually reveal the mathematical structure and reveal the mathematics. So what we draw attention to, when children create those representations, is really significant. So it's significant for a teacher to do it when they're working with their students, it's also significant for a facilitator to do it when they're working with teachers. And it's significant for us who work with facilitators to do that, too. So some of the invisible actions and interactions that we've been trying to focus in on in terms of facilitation -- there's been a few in particular we've been focusing on. One has been building moral imperative. So I'll speak to it kind of from two perspectives; one is just from the mathematics. So we've been talking a lot about students and those who struggle with math, and what we can do to help them. And a lot of times, our students -- in particular I'm going to say students with learning disabilities -- some of them have strengths when it comes to visual spatial and fluid reasoning, so their ability to be able to visualize, to recognize patterns, to see relationships -- those are often revealed to them through concrete and digital representations. So as teachers, as they begin to kind of think about and learn about how those tools reveal the mathematics, and then they start learning the profile of some of their students -- it could be their cognitive profile, or just their learner profile, and they start to kind of see how, by using some of these tools, these children actually have a lot to show that they know and understood. So that moral imperative to be able to respond to those learner profiles is pretty significant. And I think once teachers themselves start to explore these ideas, know how tools can actually help you to visualize in your mind and help you to represent your mathematical thinking, or sometimes the actions with the tools actually reveal a lot of mathematical thinking -- how, then, we are called actually to further think about those tools, learn more about them, explore them ourselves because it's a must for us to bring those to kids, to offer those opportunities for them. So what it means is, it means that we need to do some learning, right, as educators. How do those particular tools actually reveal the math? What kinds of tasks will allow those tools to be drawn upon in a way that actually showcases relationships, will be able to provide opportunities for conceptual understanding, and even link to students making connections to procedures and developing that fluency that we know is important? That balance of concepts, understanding concepts, and becoming fluent with procedure. Well, I'm thinking of one group in particular. So we had started a journey together a few years ago, and they were studying in the area of proportional reasoning, as I mentioned, in particular ratio and rate. And there was a lot of deep learning about how, for example, the relationship between quantities when it comes to proportional reasoning, and how when one quantity changes, the other quantity must change, and how that's really fundamental and important when we talk about ratios and rates, for example. And I would say that the depth of the learning that we did, we got into concepts such as, as I mentioned, rate -- unit rate. We talked about moving from additive thinking to multiplicative thinking, and the kinds of ways we might support

students in moving in that direction. And just being able to shift the conversation from more of teacher talk to more student talk and sense making. And I would say that was a pretty big takeaway for teachers to think that part of the job is selecting a task that has the opportunity to bring out rich mathematics, but at the same time, giving children the space and time to wrestle with some of those ideas. But then how you facilitate sharing in the conversations with the class is very important. That then shifted in the second year to a more specific focus on fractions, because their relationships were starting to kind of emerge, and those connections were emerging. And I would say something that was really significant about the learning that they had around the fractions was the role of the unit fraction, how important that was to be able to talk about fractions; named fractions represent fractions, also be able to think about operations with fractions. Also, to be able to think about the complement of a fraction and how the fraction with its complement, when added, will make a whole. These were some pretty important things, and these were some very important ideas. And I would say it was highly facilitated with the use of concrete tools and digital tools. So our developed resources from the ministry, in particular the Mathease digital tools, played a significant role in having them think about those relationships; being able to visualize fractions. So when they were comparing, for example, three sevenths and three fifths, many of them were relying on that spatial reasoning, that visualization in particular, to be able to think about how both of those fractions compared to a half. So three fifths, they knew, was greater than a half. But they also knew that the three sevenths was less than a half. So that meant that three sevenths was less than three fifths. But a lot of that work came from using concrete and digital tools representing and being able to talk about how those fractions compared concretely. And that helped to kind of move it into that kind of visualization, where now they weren't relying on common denominators anymore, but they were really relying on what that meant in comparison to benchmark. So benchmarks are kind of written within the curriculum, but how we help support not only students but teachers, and even us as facilitators, and being able to recognize and see those relationships is what can make a difference. Some of the things I would love to see in a classroom that has really significant learning going on in mathematics, so I'd like to see students active in their learning. I'd like to see students drawing on learning tools; concrete tools, digital tools, to represent their thinking. I'd like for that kind of thinking to be valued by all teachers. Sometimes tools have typically been classified for students who don't understand the mathematics, or were always struggling -- they need the tools. But yet concrete and digital learning tools have the potential to actually reveal significant mathematics, mathematical structure, can enable students to really showcase some of their thinking. And sometimes, the thinking that they can do through tools actually is able to be communicated better -- I don't know if better is the right word, but can be communicated in a way that allows us to see their thinking, that doesn't always get on paper. So sometimes what happens is, you know students struggle. They're not sure. They don't write enough, they don't communicate enough. They don't put enough detail in their solution, they forget stuff. I think digital tools and concrete tools give us the opportunity to pay attention to what is not communicated verbally. What are the kinds of actions, what are they doing with tools that actually reveals significant mathematical thinking? I think it's going to be fundamentally -- that fundamentally shifts the way we think about our observation, that it's not just about what they write on paper or what they say. It is important to have kids talking about their math and thinking about their math out loud, and certainly articulating and documenting it, annotating it in paper, or digitally. But I think what we want to do is try to value, and almost leverage those kinds of representations, not only for those students who may be able to, say, do that more easily than maybe putting to paper, but how do all those things connect together? How does a numeric representation connect to a visual or concrete representation? How does that connect to what we say about it orally, or describing it? This is, I think, what will create a very rich kind of environment of learning and thinking. We want students to wrestle with ideas and kind of talk about ideas, and ask each other questions so it's not student to teacher, student to teacher, student to teacher. It might be teacher to student, but then student to student, and student to student and student to student and student to student, and then teacher to student, and then student to student. So we want to change the shift, where the talk and the conversations and the thinking lies with students. Our job as teaches is to try to tap that out of them, provide the learning opportunities, but pay attention to the discussion, the representations, because our job will be sometimes to find those moments to draw attention to something, because it's going to be a really deep learning moment. Not just for one or two students, it might be for the entire class. So for me, I think that's one of the things. So it's having representations, having kids talk, negotiate their ideas. It's about an active classroom where students are doing that, they're problem-solving, they're thinking, they're building, they're talking. They're coming up with a conjecture. They're testing it. Does it work all the time? That's the kind of classroom environment we want. And in fact, that has a very strong connection to the mathematical processes. If you think about all that I've talked about, it's about reasoning and proving, it's about representing. It's about connecting. It's about problem-solving. It's about communicating in different forms, not only in verbal forms, whether it's written or oral, but also in non-verbal forms, such as using tools, and gestures, for example. It's about reflecting, thinking about the connections. Why does that work? When does it work? Does it work all the time? So I think that engagement in the mathematical processes is really the kind of environment we want to see, because the mathematical processes allow students to develop that understanding and that fluency, and allows them to apply that learning and actually show us, after a segment of learning, what they know and understand.