Growth Mindset Thinkers
Video: Five Dimensions of Powerful Mathematical Classrooms

(ALAN SCHOENFELD:) We start with the goal of producing students who are mathematical thinkers. Which means they've learned a lot of content, they have certain habits of mind -- they're called mathematical practices or processes. They're good at using what they know flexibly, reliably, so they can grapple with mathematical problems beyond what they've just been taught.

Then the question is, What are the attributes of mathematical classrooms that produce kids who are those kinds of thinkers? And the question is, What are the attributes of classrooms that result in kids emerging from them as powerful mathematical thinkers? And it turns out there are five dimensions of powerful mathematical classrooms. If those things are working well, then the research indicates the kids really emerge as powerful mathematical thinkers. So I'm going to list them one at a time and give you a little description of each. Or the rationale for each.

The first dimension, not surprisingly, is the quality of the mathematics that gets discussed. If the math is impoverished, if it's just about applying procedures, etcetera, there is no way that the kids can emerge as powerful thinkers and sense-makers. A key part of what goes on is it's not just the mathematics and what the teacher presents. It's what sense the students make of the mathematics. And those are dimensions two through five, the true framework. So dimension one is the mathematics. If the math isn't right, the kids aren't going to emerge with it being right. But there's a lot more.

So dimension two is known as cognitive demand, which is a technical term. The real question is, Are the kids getting to do sense-making or are they engaging in what we call "productive struggle?" Here are two ways that it's not. If the math is cut up into little bite-size pieces so that the kids never get to do any real thinking, then they're not building their mathematical muscles. Or if the math is over their heads and they're given challenges they can't make sense of so they wind up just memorizing, then they're not doing the right kind of sense-making. So the real challenge for instruction with regard to that, is how does the teacher make sure that the kids are engaged in a way that they're actually doing some thinking and problem-solving within the range of what they can do so they grow by virtue of having done that sense-making? You understand something the best, you remember it the best, when you do some of that sense-making yourself. And doing that means that the teacher has to have arranged the contingencies of instruction so that the math meets you where they are, the instruction meets you where you are, and you can grow. Okay?

Now that leads me to the next dimension, which is something we call "formative assessment." To talk about that, I need to make the distinction between formative assessment and assessment as it's usually defined, which is summative assessment. That's what most parents know. That's what most administrators know. Assessment is testing. It's do what we -- it's what we do at the end of the year or at the end of the semester to find out whether the kids have learned what they should. Well, that means it provides no useful information to the kid about getting better.

Formative assessment is assessment done in the moment of instruction in the service of helping teacher and student understand what the student is understanding so that you can do something

about it. Okay? It's not assigning scores; it's looking at student-understanding in such a way that you can go, "Oh, this student is getting this. That means I have this to build on." "Oh, this student has this misconception. I need to do something about that."

Formative assessment is a broad generalization that it's a way of saying my job as a teacher is to understand student thinking well enough to anticipate what students can do, to provide students the opportunity to reveal what they know. So that instruction can meet them where they are and help them move forward. And if you do that successfully, then you're adjusting cognitive demands so that the students are doing sense-making and building their own understanding. The math is central and the way that the kids engage with it is central. They need to be engaged in productive struggle doing sense-making. And something like formative assessment helps the teacher arrange for that to happen. And that means the teacher is centrally engaged with student thinking in a responsive way. But there is more to the environment than that, and there are two more dimensions to go.

The first is basically the equity dimension. When I talk about a class being mathematically powerful or teaching robust mathematics, I mean for every student in that class. We've all been in classrooms where the mathematics is beautiful, dimension one is just fine, because the teacher has picked on the three favourite kids who get the mathematics and called on them. And the mathematics is beautiful. And the rest of the kids are off in space or lost or confused. From my point of view, that classroom is a mathematical failure. Because when I talk about producing powerful mathematical thinkers, what I mean is enabling every single kid in the classroom to engage powerfully with the content. To do sense-making with the content of the classroom. So the equity dimension or access to powerful mathematics doesn't imply just to a subset, it applies to all of the kids in the classroom. And that says something about the tasks involved. Because different kids come to different tasks with different strengths, with different understandings. And if you use narrow-band tasks, tasks that you can only approach one way, that depend on specific band of knowledge, you're actually handicapping yourself. You're limiting the mathematical riches you can get out of the task and you're limiting the kinds of conversations that you can have. And the numbers of ways that kids can sink their teeth into the tasks.

So a lot of what we talk about with regard to tasks is about tasks that have low floors and high ceilings. They're easy to begin to understand. You can make progress on them in relatively unsophisticated ways, but also in very sophisticated ways. And then if you begin to link those ways in classroom dialogue, students can make pretty interesting connections.

Jargon terms for the name, the last dimension is called agency, authority, and identity. The question being, Do kids develop a sense of agency in mathematics? Do they come to see themselves as people who can do mathematics? Do they develop a sense of themselves as math people? I'm someone who can sit down, and confronted with a mathematical challenge, can do it.

So I'll start with a story that every math teacher will resonate to. You're at a party; someone comes up and says, "What do you do?" And you say, "I'm a math teacher" or "I'm a math person." And the immediate response is, "I'm not a math person. Math never made any sense and I'm not a math person." So that's that person's mathematical identity. That person developed that identity in math class.

So what we have to recognize is that part of what people walk out of math class with is their sense of who they are and what they can do as a math person. And that means that an important dimension of mathematical classrooms is, What opportunities do those classrooms present students to build the opportunities to justifiably see themselves as people who can do mathematics and explain mathematics? So that they come to see themselves, and others come to see them, as legitimately robust understanders, robust performers of mathematics. That's what constitutes building a powerful mathematical identity.

A very serious question for any math classroom is, When are kids having the opportunity not only to do sense-making, but to talk to each other about it, to make contributions to the classroom discourse, to have others build on their ideas so that math class becomes a place where people sort out reasoning collectively, build on each other's ideas? And as such, come to engage productively in mathematical struggle and in sense-making? Those are the conditions for agency, authority, and identity. What you want is people emerging from mathematical classrooms, one, with a rich sense of mathematics. Two, with the ability to be flexible thinkers in mathematics. Three, with the willingness to sink their teeth into that, and to persevere at that.

And now if you think about the dimensions, dimension one is the mathematics. It's got to be right. Dimension two is cognitive demand. The kids have to be positioned so they're doing sense-making. Dimension three is formative assessment. It takes a lot of work on the part of the teacher and lesson-designers to set things up so the kids can be engaged productively. Dimension four is access and equity. It's not for the chosen few; it's for everybody in the classroom to sink their teeth into in the right ways. And if they have the right opportunities, then they get to build productive identities for themselves as people who can do mathematics. And the evidence is that environments that have those attributes do indeed produce kids who are powerful mathematical thinkers.