They think that we need the right kind of math. What about pedagogy. What about pedagogy in the early years. Part of my work in this project has been to be in the midst of debates right here in Ontario about what young children should be doing and what kind of pedagogy we would be talking about. And there seems to be quite a break in thinking between it emerges from play thus its natural, or its direct instruction and then children are not very involved. Well, it is true that very direct instruction is inviting children to be passive. They are not interactive. They're not necessarily contributing because there is an agenda about what they should learn. So that is direct instruction. And you know, emerging from play is, in its strictest sense, would mean like in the strictest sense of an inquiry, that this is, that what the children do that's mathematical is something that emerges from them as they're playing. A very interesting research study which I feel needs to be described, did they, you know, interesting job of trying to compare pedagogies, and I will just briefly outline the study and the findings and then sort of talk about the implications of that. This research comes from the world of developmental psychology. And the question that the psychologists were asking was where will children learn most. Where can they be most thoughtful in math? In particular this was a geometry, it was, it was a learn, a specific learning in geometry that they were interested in. And they wondered would play alone or direct instruction, or something intermediate which we all know in from different names, they're calling guided play, which would be the most successful approach to have young children, these are kindergarten children, be able to generalize something? And this is what they were hoping for. So in geometry we know that when we're talking purely about shape recognition and shape learning, we know that young children will consider a triangle to be only something that is pretty regular, that they see all the time that sits on the horizon that has a point on the top that looks a bit like a roof. When they would see an acute triangle most kids would say, it's a spear, it's not a triangle. So it's very much a very initial visual representation. Where we hope children will go is that they'll come to recognize that triangles all have three corners, they have three lines, that they're closed figures, and that anything with these attributes is actually a triangle. And so these researchers were hoping to move this group of children, who they interviewed one on one, from a level one approach to geometry, which is completely visual, to a more analytic one. And why this study was so interesting to me is that the guided play and the direct instruction paradigms or methods were extremely similar, except for the way the teacher questioned the children or talked to the children. So in one, one group of children, the children who are in the guided play, this is what they would hear. They would hear that, and they would hear that I'm practically quoting from the article now, "All triangles and all shapes have secrets about what makes them a true shape. And I want you to look at all these triangles and see what you can tell me about those triangles, because they do have secrets. Oh you notice that they have three sides. You notice they have three points. Here is the direct instruction." And in both cases, which is so charming, is that the experimenter or the teacher was wearing like a wizard's hat just to make it very playful, and smiled, and was just as inviting with the children. But in this case she, what the researcher said was "All triangles have secrets that make them triangles. I'm going to tell you what they are." And then just proceeds to tell them, three lines, three points. And then the free, the emerging from play or the totally free play with
no guidance was just what we might imagine. A lot of triangles a lot of different ways of playing with them. A lot of ways of making triangles. They were all very parallel, these conditions. By a very huge, by a huge amount, not exactly scientific, but a significant difference in terms of children being able to generalize what shapes where, was in the group that were told what to look at about the triangles, to look and then to explain what they're seeing versus to be told. That was huge in the play only condition the children didn't make any gains. But of course this was a one shot, there's no play, it doesn't work like that, so we don't even have to consider that. It was just the nuance of the difference, just that maybe we have to rethink what direct instruction looks like. Perhaps if somebody had come in and they'd seen a bunch of children sitting in a circle, they were in kindergarten, and they were being directed to look at these different triangles and directed to think about what the attributes were, that might have been considered. We can't do that. That is like math teaching. But it was so close to what maybe is not the most successful way to math teach which is really to tell children what it is that's happening. So we need to invite children. The overall, I mean we, I think we know this as math educators, but I think a big message from that was just how important the children's role is in the learning. How important is that that they are active participants. That they are the ones who are, who are offering what they see. They are the ones who are doing the building of the triangles. So I really worry that we're still in a kind of messy state about pedagogy. That we have images of what's bad and what's good. That these need to be rethought. That we have to think about pulling these together. One of the, again, lessons of working in these different communities was just how long children were interested to sit in the circle and talk about what they think about the structure they've just built, the pentomino shapes that they are now finding, what they're coming to realize congruence means, what they're coming to realize congruence means, what are transformations.

MORE MATH

>> In terms of recommendations, I feel like one of the things that is really been a learning from this project is to think about how we can have more math, not just that we can have math but how math can be part of the full day in the early years. Math involves literacy, math involves problem solving, math involves a huge stretch on, of creativity; math is particularly spatial reasoning math is part of art. And so. If one were to say, what could we be doing change the curriculum, I would say, 'we just need to have more of it' and we have to understand that just the way we understand that literacy is something that is across all subjects that the children will be doing, math has the potential for that as well. Why do we care about math in the early years? It's shocking to hear this but, math anxiety is something that is already part of children's lives by the time they are in Grade 1. We know from research that if children in Grade 1 have a teacher who is math anxious and if that teacher is a woman and they're a girl, they are going to be math anxious by the end of the year. How do they research this? They look at children's attitudes before they start Grade 1, they look at them at the end of Grade 1 and they can see that children are already math anxious. There's a huge issue of identity around math and this is something that we all know that is pervasive. Identity around math, you talk to adults and they're quick to say, 'Oh no math is not my thing, I'm not good at math'. But when you hear a kindergarten child say, 'I'm not good at math, I
hate math', how is that possible? So this is something that we know that does not have
to be. So we need more time in math, we need math that's compelling, math that is not
static, math that takes account of children who are not yet necessarily strong in number;
because the kind of math they're doing is the work of mathematicians. And I think that.
We know that children's play, spontaneous play has lots of mathematical elements;
they're building, they're looking at measurement, they're patterning, they're doing
Algebra, they don't know that's math; that is not what they think math is. Math is a very
special little part of the day where you either are pretty good at number or you're not yet
there at number. And if you're not, then you're already starting to feel behind.

MATH KNOWLEDGE FOR TEACHING

>> If I were to try and really continue to think about what would make an impact in early
years Math, I would also rethink as a Math educator what mathematical knowledge for
teaching is for early years and how that compares to what the field has, very, very
strongly defined the upper ball. And others have given the research community in math
education very powerful tools to analyse the degree to which a teacher has
mathematical knowledge for teaching. Of course, it's not only about knowing the math,
knowing your children, understanding pedagogy, putting it together but very specifically;
it is looking at how teachers understand and would be able to teach fractions,
proportional reasoning, Algebra, data management. This is these vignettes of what it is
that teachers can or cannot do yet, are very, very useful when we are teaching our
teacher candidates as I do our M.A. students and looking at their knowledge. But I think
that the knowledge for teaching in early years is not something that we have yet found
and yet been able to really claim that we know enough about. And I think that would be
something that would be very important as the next step to think about where we're
heading with math in the early years. What is authentic? And there is such a, usually
the word authentic math is used in conjunction with real world math and questions of
well, okay, so the children can do this in the classroom. What does that have to do with
their daily lives? Their future and having been fortunate to be observing children doing
math for maybe 20 years now, children as young as four and three and as old as sixth
grade, what I've come to see is that what's authentic, what authentic means to me is
that it has a purpose for the children that's meaningful, that it's challenging. Playing with
numbers is just as authentic, an experience as playing at a cash register and figuring
out how those numbers relate to buying food in the store. The idea that ten combines in
so many different ways and I think that is as meaningful to children as trying to imagine
that their taking their pennies now to the store and spending their pennies. I think that
we don't give children the credit that's due to them to be just genuinely excited by how
shapes come together. How shapes come apart. How things are symmetrical. In just
yesterday in a classroom we were working on symmetry, very young children and the
fascination with what happens on two sides of line of symmetry. The intuitions they
have for it, the knowledge they have and you know is quite remarkable. They don't
have a definition for it but they do know from, just from their intuitions. They do know
that the two sides on either side of the line match and that they are mirror images of
each other. And yesterday's class, that they're, who used the word congruent in
kindergarten, that they're congruent. There's a huge amount of math that is extremely
satisfying for children that's not necessarily something that is directly applicable. And so I feel that we have to think more about that as well, what makes something authentic? We certainly know what makes P.D. or professional development authentic for teachers and the teachers in our project knowing that they are designing brand new tasks, have this feeling that they are actually making a contribution and it's genuine. And that is what sustains the work that we do and I believe that kids, children, young children can engage in math and with authentic purpose.

MATH FOR YOUNG CHILDREN

>> Four years ago Cathy Bruce and I started a project called 'Math for Young Children'. Working in collaboration with the Literacy Numeracy Secretariat, the project was, the goal of the project was to look at math in kindergarten classrooms and to think about how to enrich math for all children who were starting full day kindergarten. The motivation for this work came from a Lit review; a literature review that Cathy Bruce and I conducted in advance of our work in classrooms. What we learned in that literature review was very new for both of us, as part of this Lit review we learned about recent findings about the importance of math in early years. In 2007, economist Duncan and many colleagues from Northwestern University looked across many, many years of studies of children's math success, it's and general success in schools; what they found was that, what children do in kindergarten in math predicts what they are going to do not only in math but also in all schools subjects by Grade 6. This is surprising! We would have imagined that it was how they performed in literacy in the early years that would predict how they would do generally in academics. In fact, you know, literacy is something that is very much privileged over math in early years. In fact math is even, the teaching of math is even slightly controversial in early years; not considered necessarily appropriate and so the literature review and the follow-ups and what else we learnt about play and the potential of play but also the limitations of learning math directly from play made us really try to with teachers with early years educators rethink what we were going to be doing with young children. The 'Math for Young Children' project has taken us to very culturally diverse communities. We've worked in schools of very high resource communities. We've worked in First Nations communities. We've worked in schools highly English as a second language base. And we've worked in schools where the community has problems with poverty with the goal of learning what is important to children. One of the things we learned in our Lit Review is that over the years there's been a pendulum swing around, you know, the, about shall we be teaching math? What kind of math? Should math emerge from play? Should we have no math at all? Is math inappropriate for young children? Is it not a time for promoting the social-emotional needs of children for looking at relationships for thinking about literacy? Why do we need to think about math in the early years? And I think that there hasn't been any definitive answers to that but I feel that we are now coming up to some definitive answers. If we know that math in kindergarten predicts later school success even more than does literacy, we have to really think about how are we going to promote math in the early years. What is it that young children can do that's meaningful to them, that's authentic, that is challenging, that is rigorous, that's mathematical. Usually when we think about math in kindergarten, we think about
number, we think about counting, it's very, very important that kids count. But
unfortunately our image of learning through number is usually one that's pretty static,
that sometimes even frighteningly involves paper and pencil, matching one number to a
number, a numeral to number of objects. And I feel that, that's the kind of image that is
one that would make proponents of a play-based approach to math, you know, that
would really make sense if that is what we see math. It is so much bigger of a topic
than number and in the way that we prioritize literacy in early years. We prioritize
number as a way of thinking about what young children should be doing in math. As
part of the 'Math for Young Children' project we conducted a small survey to look at
what teachers then JK, SK and Grade 1, we asked them to rank the topics in the math
curriculum in terms of the amount of time they would give to that subject. What we
learnt from that is of course number was first but in kindergarten of the five topics,
geometry was the fourth and in Grade 1, Geometry was the fifth. When we read the
research and we hear that there's a predominance of number being taught in
kindergarten, we actually discovered that was true right here in Ontario. As we
embarked on the 'Math for Young Children' project, one of our goals was to look at
areas of the curriculum that are much less practised by teachers, number would be first
and certainly patterning to Algebra would be second. And we decided that we would
look at spatial reasoning. This was also the beginning of our, as a research team, our
learning about spatial reasoning, the importance of spatial reasoning.

MATH BEYOND NUMBERS

>> Math should be beyond number. And you know, in starting this project when it goes
to all kinds of different sources. And it was very interesting for me to see that the
National Council of Teachers of Mathematics in the recommendations for what children
should be doing in kindergarten, grade 1, grade 2. Suggest that half of the time in class
in the math curriculum should be spatial and orientation. Should be geometry, should
be measurement. And, indeed, we have learned that wouldn't be something that would
be followed. That it's something that we have to offer. I think, again, I think we're in, I
feel like we're in flux of learning. When we're teaching literacy, we don't think about, oh,
we've got to finish a whole unit on sound. No, we know that we want to infuse all
different kinds of aspects of literacy in a very balanced way. I think that is something
that teachers and we all naturally understand. So you wouldn't be focussing on
questioning one day and know that you'd had the questioning part or you wouldn't be
focussed on, you know, writing particular kinds of sentences. And I think that what
happens in math and mathematics teaching is that we have a sense that, well we have
a trajectory, we have a unit, we're going to go from A to wherever we need to stop at our
grade level. What the vision would be that we would come to understand the teaching
of math, the way we do literacy. That we would know that we could integrate the
learning of number with the learning of symmetry. That geometry and number can
come together. That we would be thinking about data management in terms of its
spatial aspects. And we know that kids can see when they've made little graphs of
whose teeth have fallen out and how many teeth have fallen out in the class. They can
actually see the differences, the similarities. They can see trends all visually and
spatially. I see that there needs to be a balance, but that this is not, it's not a matter of
saying it's the month of January. We're doing geometry. It's the month of February, we're doing measurement. But somehow that these come together and these be many of those different topics be part of a full school day. Terrible answer, but, I mean, that's a general gist of what I actually think. But maybe I could answer that another time because I haven't ever sort of written about that. And I guess as the group of our math-free young children group of researchers, as we talk about what we keep saying spatializing the curriculum. Maybe that would be an overarching idea that brings together the different strands. And, certainly, in early years classrooms. And I'm up to grade 2 when I'm saying early years. I think that this approach of integrating the different topics is something that I would recommend if we were ever to build similarities. That is what emerging from play really means, right? Because you're not dictating that the kids do patterning as they build with their blocks. Or that they do measurement. Or measurement of height or measurement of weight as they negotiate the building. We are not thinking about, okay, well, now it's geometry as they are thinking about all the transformations as they're looking at symmetries. As they're looking at balance. We don't think about it that way in their play. And so if it emerges from play, basically, all of these ideas are emerging. What would be the beautiful goal would be that this would be more planful and that there would be a way. Not necessarily of naming that this is the strand we're now working on, but knowing how they come together. And maybe this would be part of what it means to have knowledge of math for teaching as an early years person. Part of what is called upon as an early years teacher is, you know, a deep understanding of development. With I think most teachers that would be something that would be very important to them. And the added understanding of the development of the different topics that they're teaching. And there are trajectories. We know Doug Clements and Julie Sarama have done just a brilliant job in bringing us stories of development. Very, very specific ones. How children compose and decompose 2 dimensional shapes. How they, the development of their recognition of shapes. Development of early number. And these are helpful. But teachers need to spend more time personally engaging, I think, in all of these activities. Rather than looking at a trajectory on a page that is not necessarily going to inspire how they look at their children.

CONDITIONS FOR EFFECTIVE TEACHING

>> Well, you're saying the belief that children can learn, maybe the knowledge just from a teacher's perspective, the knowledge of the importance of math and the knowledge that, by engaging in math, children are also engaging in -- as they work in groups in their social emotional development, that they are -- that it's a different opening for discourse, literacy, talking, language development. These are all part of teaching math. Teachers maybe need to have some new images of what math teaching might look like. And so, when I say that, in a paper by Herb Ginsburg talks about the kinds of misconceptions that teachers of young children have around math teaching and or the kinds of -- maybe it's barriers. That's more what it is. It would be barriers. Why is there not more math? Certainly, one of the problems is that people who choose to teach in the early years sometimes will say, I have done early years because really I don't -- I really don't like math, and I don't have to teach much math; or I'm a little math anxious.
So that's what I'm saying about new images of what math teaching is and can be. And that -- so I guess it's the importance. It's a space for math, and it is maybe changed attitude about or changed understanding of what doing math is and understanding that there's -- yeah. So I think there's that. I know that in this list of misconceptions it's also a huge one about that it should only be play based. I think that there is a concern -- a concern by many who themselves are not comfortable with math that why would they impose something on their young children when it's not something that they're comfortable with. There's just a huge world of math, and maybe -- I mean, I'm just thinking about how I feel that teacher education programs need to be more proactive in terms of what it is that they are helping perspective teachers learn about what they might be doing in a classroom. If we know that it is having children interacting and being active as they do math, this is a condition, that the ability of teachers to expect that their children may have many mathematical intuitions and ideas that should be listened to, I guess this is a condition. The absence of a notion that everything has to be done quickly, I mean, more and more we are learning in our field about how detrimental timed tests are and how that causes anxiety. And mathematicians are kind of surprised that that would be a number one focus as kids get older. It's how quickly they do it. So I think that allowing kids to be slow and being able to recognize what is emerging as children engage in activities. As a project, we knew that our mandate was to find out what's possible. We were thrilled by what we saw young children being able to do. We did learn that they could work for extended periods on problems that their teachers had just worked on as adults sitting at a table and wondering. When we brought our early findings literally in the form of videotapes of children working in classrooms, we were met with but that's teaching math. We don't teach math. You cannot -- children are sitting in a circle, and they're having a math lesson. That's teaching math. I was very shocked and hurt. And I'm saying, you know, that's the intimate place for children is to sit in a circle and listen to a story and discuss their emotional reaction to a story, to analyse the characters' motives, to talk about something that happened at home. And here are children who are discussing their emerging theories about why something is -- I repeat this example but why two objects are actually congruent, but it would be hard to know that unless you rotated one and then you could see that they were the same. There's just a lot that young children have to offer. Also, what sitting in the circle -- maybe this is an important lesson -- offers as well is the idea that maybe something that people would find concerning, would they be copying, is that all children can see each other's work and build upon each other's work. That's something else that we were able to show and model. And so it has been with a lot of thoughtfulness that we have moved forward and to try and promote the potential of young children by just bringing pictures and new images of what doing math in early years might look like. For conditions to work, we need more people in math education who have been part of the math education community to think about how to be involved in math in early years. So this was another -- this is another bit of a hurdle because math educators start usually thinking about high school, and then a handful of them are thinking about elementary. But very few bring their expertise and knowledge of the development of math into the early years. People who work in early years tend to be experts in early years but not necessarily have an expertise in math. And so that is something that would need to happen in order to move forward. That's one of the conditions.
A STRONG LEARNING ENVIRONMENT

Another lesson from research from Megan Franke's work in Los Angeles, you know, resonates here. Although this wasn't about such young children. Megan Franke, who is a very, very, very, prominent figure in our math education world, very passionate about trying to make a difference and seeing what happens mostly in situations where children, well we'd say low-resource communities, where probably test scores are low. Looking across all these classrooms who were involved in a significant amount of learning, the teachers were, and wondering what as maybe -- what was unique to the success of children in these classrooms? And I think the feature that was number one was the amount of listening that the teacher did to her students. Not only that, but the amount of time that children were given to explain their thinking verbally or other means but encouraged to go beyond a first explanation what makes a really strong learning environment. And as we move from classroom to classroom, I think we are also seeing that in classrooms where teachers encourage and allow students to express their emerging mathematical ideas and to explain their ideas and to explain why things don't work, I think that's another huge lesson from our project. We're also seeing that. That is the most supportive of environments. We could see that even in the clinical interviews because teachers don't often have the opportunity to sit one-on-one and just wonder. I wonder how a child will respond to this question. I wonder how they'll do. I wonder if they'll come to see that two shapes are the same even though they're oriented differently. It is in the -- in working with children in this kind of way that children are given a kind of voice that I think all teachers would like them to have but don't necessarily see a route to that in early math. Children have a huge, like a huge store of intuitions about mathematics that is ultimately going to be very sophisticated and formal. So children as early as kindergarten can reason about fractional relationships, about proportional relationships, the part that -- the damper on that is that we often try to add the more formal number aspect to these intuitions. The kind of math that maybe it shouldn't be so number focussed when we do math. That the pedagogy is warm, is encouraging, and it's guided. But it's guided by ideas that the teachers have about what it is that the children should be doing. This wouldn't be different than what we would recommend in grade three, in grade four, and further up the grades.

PAYING ATTENTION TO SPATIAL REASONING

What we realize although that is one area of mathematics that would call for spatial reasoning is that overall it is a topic that is pretty static when we think about the developmental levels that are offered to the community and to teachers. In geometry these are the levels by Van Hiele's and what they're suggesting is a very analytic and static kind of developmental trajectory. In our goals to combine what we understood about the importance of spatial reasoning and the importance of broadening the curriculum in math, to include, a much broader range of topics; we worked with teachers to think about how we could, what we've been calling, 'spatialize' the geometry curriculum, that led us to thinking about activities that involve much more building, much more three-dimensional thinking, much more moving from two to three dimensions.
Much more focus on involving the children in visualizing, in mentally rotating both of which are considered to be key for success in mathematics. So our focus on spatial reasoning came from a variety of different fields from cognitive science, from neuroscience and from math education. And so the work that we've been doing; the resources that we've been creating with teachers have a strong focus on spatial reasoning. As we start with the focus on spatial reasoning as researchers, what we learn, as all of us having been teachers and sitting in our classrooms of kindergarten children, Grade 1 children in these wide varieties of communities is that these kinds of ways of thinking, these ways of interacting with geometry that are maybe often part of the Grade 4, 5 and 6 curriculum is something that the children find compelling, find exciting and put a lot of effort into. These, so as the teachers who we are surrounded with and a lot this is true in all of our communities but maybe especially in our First Nations communities; is that, starting from a spatial orientation these are very much hands-on activities, they involve lots of construction, lots of dissection, lots of drawing that students who, that all students seem to be excited, this seems to be a way in that has multiple entry points, that allows all students to participate and so we are continuing to study the effect of spatial curricula in the early years. We watch children engage with geometry and mathematical activities in ways that are sustained, in ways that are exciting, in ways that seem to be appropriate for all children. But we've also had the opportunity to look at outcomes when children engage in these kinds of activities. What we learned from our work in Fort Frances, is that, children whose teachers work with them and these were all the teachers who work with them on activities that are spatial. Some of them are some of the activities that the children engage in are long kinds of lessons with beginning, middle and end. But a lot of the work that we are thinking about for these children devising and playing with are quick image activities; and our findings are that children who have lot of experience, over in the case of our research; so far a four or five month period, not only make huge gains in geometry in their spatial language, in their ability to name shapes, in their ability to work with location, in their ability to do very, very difficult and challenging mental rotations. The children also make progress in number even though the curriculum really is very much one that is focussed on space. And this research was carried out and compared to a control group of students who were very, also doing a lot of professional development but doing it in numbers. So we're feeling from this study and all our current studies that we're going to continue working in this area. The spatial reasoning document comes from a lot from the work of math for young children. It was the people on our team who actually, we brought these ideas and I think that, you know, we've talked a lot about math as a predictor to later school success and therefore needing more math. But we haven't necessarily or haven't necessarily mentioned the-- how spatial reasoning and the ability to spatial reason which and the ability to in particular visualize, be able to rotate mentally these are qualities and reasoning abilities that are underneath and predict all of peoples' work in the stem disciplines and what's one of the things from the perspective of early years that we're learning and that we do know is that children as they get older, they maybe don't chose to think about math, engineering, technology, architecture as potential avenues because they don't feel confident about their spatial reasoning. By starting very early and by supporting young children who are surprisingly adept at spatial reasoning; surprisingly, this is not an area that's been studied as much as spatial reasoning in adults and older children. What's inspiring about the literature in spatial
reasoning is, study after study after study shows that spatial reasoning can be improved, right? I think that there's an idea that spatial reasoning is something, it's a gift, you have it or you don't have it. Many people, many adults, the same way as saying, 'I'm not good at math' are also very, very quick to say, 'I'm not good at spatial reasoning' and with the understanding that, that's just something they have a gift or not. So we have learnt from meta-analyses, from individual studies that spatial reasoning can be improved in all ages and interestingly; people who start off with a strong spatial reasoning sense, they will improve with training. People who are lower, it takes longer but then, they improve and they move very, very quickly after an initial sort of hump in that sort of more eloquently shown in graphs. And so by giving children the opportunity to feel a strength in spatial reasoning in the early years we are hoping that we're setting them up for making choices, choosing math, choosing science, choosing engineering, choosing architecture because they're already invested in spatial reasoning. What we're promoting in citizens, what we hope people will come away from schooling with is, a comfort with math, a sense that there's some way I can figure out that problem. I can look at that graph and I can actually interpret it.

IN THE HEAD OF PRACTICE

>> The first two years of math for young children, I think we can find out exactly. But I think maybe we were, we had 14 learning communities. It was a lot of learning communities. We went in as co-designers, as researchers. We introduced our community of learners to literature that had inspired us. This was work on the question of early years and the potential of early years math to make a difference in terms of later success. We looked at some of the more vibrant and compelling literature about spatial reasoning. We had discussions with our learning teams, our professional learning teams. And in the two years starting with pretty much of a model of Japanese lesson study where we ended our session with what's called a research lesson. We put together all our learning, we invited lots of people to come and watch this research lesson. But on the way to the research lesson, what might have been different is that we did a lot of exploration. We allowed teachers to become researchers in a very authentic way, having read the literature on what's possible. Having then also introduced and a lot of geometry and measurement activities that were new to our teacher population that were the same kinds of activities that I teach in my MA class when I'm trying to support the teachers in the class to learn more math. We engaged in these activities. And then based on this engagement and based on thinking of geometry with totally new lens of something that is much more dynamic, much more creative, much more playful, and more challenging. The teachers began to ask, what can our children do? Can our children engage with these tasks? What would they understand about congruence? How would they be able to imagine an object from different perspectives? Would they be able to mentally rotate something to be able to say that all you need to do with that piece is turn it, flip it, and it's going to fit in the slot? All of these very challenging activities for teachers. And it was in the spirit of our own inquiry to our own learning about these tasks. And then saying, well, let's one on one look at children and see how they do on these tasks. I have to say that the majority of lessons and the majority of activities came directly from watching children one on one.
So this was a privilege, this was a project that had a beautiful seven full days of release time. This was a project where we were all in a way really co-researchers because this was so new. You could not go to the Internet and find lessons for kindergarten that involved geometry with a spatial focus. We had to design these lessons. And so in the first two years, we carried on in the Japanese lesson study tradition where we ended up with a lesson which became a resource across Ontario very quickly. Activities that became resources. What we called clinical interviews or teacher-led interviews that were also very shareable. And it was these two years of working in so many different settings with the goal of creating thoughtful, rigorous, challenging math that we could see that was appropriate for young children. Because it was all done in the right in the sort of heart of the practice. So two years later, where we've moved to Fort Frances and we now have a lot to offer, a lot that's been designed by other teachers. And our professional development then changes. And we now are able to offer these activities that then get redesigned and co-designed. And by our new teaching groups, our new professional learning teams, in their context, added to our work in the classroom in our Fort Frances classrooms and in the communities to work with. We knew the importance of moving not only outside of the classroom to involve families. And so this work now involves regular family math nights. Again, because of the nature of the kind of math that the children are doing, it's challenging and it can involve children from the junior kindergarten or the first year of FDK and their parents and their grandparents and us math educators. So we continue to grow this work, to think about this work. As researchers, though, we also want to know about the outcomes. We know that we know just looking and from case study to case study of classrooms that the children are enjoying this. That the teachers are feeling like it's a new way, this is the most, maybe one of the most important learnings is that the teachers have all said that they're looking at their children in a new way. They're certainly looking at math in a new way. They have expanded what, you know, our work is helping us all know how to move beyond a focus purely on number.

CULTURALLY RESPONSIVE MATHEMATICS

>> Culturally responsive in terms of really hearing your students where they come from, what they have to offer, the different ways that their parents say do long division because they come from countries where it's taught differently, culturally responsive. I think that it pertains to mathematics teaching as much as it pertains to teaching all the other subjects that children are going to learn. Culturally responsive has to do with, to me, really has to do with acceptance and an awareness and an acknowledgement that where people come from and the way that they have been taught at home is important. It's important to the classroom. It's important to the teacher. And to me, that's where cultural responsive teaching comes. It's interesting, in our First Nations' work that what we're hearing from the community, from elders is that what we're doing and how we're engaging kids in very spacial ways in building, that's not necessarily, you know, number first, this is math, has opened a door for their children. That they see as very powerful. That's a kind of -- and we perhaps had some intuitions about that, but to have that reinforced by hearing from the community and by watching the activities that we've designed being taken up by the education counsellors on the reserves and brought to
daycares, that makes us think about that this is a kind of cultural responsiveness. But maybe the most culturally responsive aspect of our work is that we have helped teachers see the children in their classrooms who might not have participated as fully in some aspects of the math program now be fully participating members because we've provided a new way for children to interact that obviously was culturally responsive as well. So that is interesting. Lisa Lunney Borden, who is one of our leaders in thinking about teaching in aboriginal communities, has talked a lot about the language being verb-based. So being aware of the language and its verb-based aspects. It's been interesting to hear the people who are teaching language talk about how the work that we're doing seems to be very aligned with the language, because it is very action-based. Although it still is very mathematical. I think a final thing that perhaps I haven't said, and this is something that I have over the years come to understand more and more. There is an understanding, perhaps misunderstanding, that there's a hierarchy in math teaching and learning. And that the high, the highest level of mathematical thinking is number and often decontextualized and that reasoning through images or through constructions is on the way to abstraction but it's not abstraction. So in my work on patterning and algebra that has become well known through Kathy Bruce and Ruth Beedy, looking at very young children being able to see the structure of linear relations by looking at growing patterns that have asymmetry to them and being able to reason based on these visual representations and their spacial intuitions that, oh, I do see that this is times 2 plus 1 pattern and that if we went to, you know, the tenth position or the 100th position, it would just be 100 times 2 plus 1. And to learn from my colleagues who are mathematicians and math educators the power of that kind of reasoning that is not what one would think of as a high level of abstraction because it's reasoning. It's reasoning through images. In rational number, I spent a lot of time having children compare relations. Oh, 50% of this huge beaker 50% full is this much, but it's the same relation as 50% full of the smaller beaker to its partner. To be able to make generalizations that do not necessarily involve number is an extremely important aspect of math that we have to really come to see, recognize, appreciate, and celebrate, because this is as mathematical as being able to do algebraic equations and maybe even more, because we're involving what's key and important is the generalization of structure. So I guess if we're thinking about culturally relevant pedagogy. We're thinking about where are the generalizations, what does abstraction look like. Usually culturally relevant pedagogy would imply that it's something that's concrete, that we have built something that is appropriate to our culture, and we've learnt all about measurement through doing that. So it would be doing that and then making a bridge to a generalization of what it has meant to do that, what the mathematics is. It's really about a sensitivity to your surroundings, that's what it really is. It starts there, listening to children in every opportunity you can, listening to families. And I think as educators of young children, that's something that we all do. But being able to hear what it is that families value and understand how that can be part of your teaching.