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A Forum for Action - Effective Practices in Mathematics Education

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What I have learned in my research

The central focus of my research over the past seven years has been on the use of dynamic geometry software at the primary school level. This focus was initially motivated by research findings indicating that children come to school with a great deal of informal geometric understanding that is not being developed formally in school at an early enough age. My choice of dynamic geometry software was based on the strong research evidence showing that it improves student learning and motivation in the middle and high school grades. I have conducted a large number of classroom intervention studies in different elementary schools, primarily in British Columbia, focussing on the following geometry topics: symmetry, tessellations, 2d shape identification and construction, angles and parallel/perpendicular lines. My research has shown that, with the help of well-designed tasks and teacher's promotion of mathematical reasoning, *Sketchpad* can help children very effectively develop these concepts, which are usually saved for later grades, and avoid several misconceptions reported in the literature. More precisely, I have found that:

- As young as kindergarten, children can move from identifying triangles and squares by comparing them with prototypical images to using their invariant properties ("as long as it has three sides").
- Again, as young as kindergarten, children can develop a sense of symmetry that moves form the unary (thinking of symmetry as a property of a given shape, like a heart) to binary and functional (thinking of symmetry as a transformation of one shape to another so that, for example, the distance to the line of symmetry is the same).
- As young as kindergarten, students can work with the concept of angle-as-turn in order to develop benchmark angles (1/2 turn, a whole turn) and navigate successfully on the plane. Further, children then use the concept of angles in describing the properties of planar shapes.
- Students as young as Grade 1 can formulate definitions-in-action for parallel and perpendicular lines. They can then use these properties to describe and compare planar shapes.
- Students as young as Grade 2 can engage in geometric argumentation in order to, for example: explain why regular heptagons cannot tessellate the plane; explain why a construction of a triangle in which two sides are radii of a circle is isosceles; explain why an equilateral triangle cannot be scalene; explain why a given figure is not symmetric, and so on.

Why might such topics be considered worth learning in primary school? There are at least two reasons. The first, mentioned above, is that children come to school with some informal understanding of each one of these topics. Second, each of these topics involves important geometric ideas that provide continuity with higher levels of the curriculum. Third, these topics all involve opportunities to engagement in reasoning and not just learning vocabulary (the names of shapes).

For the past few years, I have been involved in a research project (with Joan Moss and Cathy Bruce) that seeks to promote spatial reasoning in the curriculum. One obvious way to do so is to place more emphasis on geometry in the curriculum, which requires identifying geometric concepts that are appropriate at this age, and worth knowing. As I've articulated in my NCTM co-authored books Essential Understandings in Geometry (2011), a central aspect of geometry—and thus of what is 'worth knowing'—is the study of invariance. An elementary school curriculum should thus be based on concepts and associated tools that can help children appreciate invariance.

More recently, I have developed an iPad application (*TouchCounts*) aimed at developing number sense in learners 3-8 years of age. *TouchCounts* puts fingers and gestures at the heart of counting and adding and seeks specifically to develop the following aspects of number sense: the one-to-one principle, the cardinality principle and the stable order principle. While mature number sense is said to develop only by age 8, early findings suggest that *TouchCounts*, in concert with a teacher and the use of certain prompts, can help develop this number sense at a younger age. In particular, it helps children reify numbers (move from thinking about numbers as a sequence of words, as in rote counting, to thinking of a number relationally and as an attribute of a group), which enables them to develop a discourse on numbers as objects, which is essential for recognising the cardinality principle and for being able to work with addition, decomposition and subtraction.

Both these research strands are based on theories of learning that are non-dualist, that is, that do not assume a distinction between body and mind and, therefore, that accord a central role of the body and its senses in mathematical thinking. From this point of view, mathematics can be seen as a way of perceiving and acting with respect to various symbols and notations resulting from complex historical traditions. A wide range of bodily activities, such as talk and writing, but also gesture, rhythm and posture express and constitute these perceptuo-motor ways. Well-designed digital technologies can help children learn and express themselves in multi-modal ways that do not depend solely on written communication, and that may thus invite a broader range of ways of thinking.

What are the strengths and challenges of using technology in the mathematics classroom?

The two major challenges in using effective digital technologies in the mathematics classroom are: (1) lack of adequate professional development and (2) concept narrowness in the curriculum.

In my work in schools and with both preservice and inservice teachers, it is clear that teachers need adequate support not just in accessing digital technology, but in learning how to use it purposefully. I have found that primary teachers want to use digital technologies (unlike many secondary teachers, who do not have a culture of manipulatives in their classrooms), but (a) are not familiar with effective resources and so do not make "purposeful use of technology", as a Grade 3 teacher in Alberta put it; (b) do not necessarily know what the important mathematical ideas associated with a given task might be, and so have difficulty identifying appropriate problems, asking effective questions and knowing what to assess. These challenges are very well known to the mathematics education research that has sought to explain the relatively low level of digital technology integration in mathematics classrooms despite widespread access.

My research takes as given that digital technologies range widely in their classroom potential and that no digital technology can be helpful in absence of a good task and a good teacher. Further, any tool that is chosen as a resource in the classroom (whether physical or virtual) has its own two-way relationship with mathematics. Because of this, I think that it is very important for curriculum documents to avoid vague allusions to "digital technologies" or even "dynamic geometry software" and to be more precise about the tool/concept/task combination. For example, symmetry as a concept in the *Sketchpad* microworld of "Symmetry Machine" draws attention to certain aspects of symmetry that are mathematically sound, but that may differ from the way symmetry arises out of paper folding. If curricula continue to be written for a default paper-and-pencil toolset, it will be very difficult for teachers to see how they can develop concepts using other tools.

What further questions about mathematics teaching and learning are you now beginning to examine?

As presage in my response to the above question, I think that the main research questions should now focus on how best to support teachers' effective use of appropriate digital technologies. Very little research on this question, especially at the elementary school level, currently exists. At the risk of sounding too hopeful, I think that the availability of touchscreen devices may prove to be a tipping point in teachers' adoption of digital technologies, given the distinct advantages over desktop computers that they offer, both in terms of hardware (easy to integrate with other classroom resources in terms of sharing and mobility, and software (applications tend to have a very short learning curve and enable non-written forms of communication). Since there currently exist a multitude of applications that favour procedural, level-driven mathematical thinking, teachers will need help in (1) identifying apps suitable for problem-solving, discussion and exploration, (2) knowing how to orchestrate digital interactions with physical ones (both with manipulatives and with symbolic, paper-based media), (3) find ways of using applications to conduct formative assessment.

Based on my work with inservice elementary school teachers, providing them with opportunities to watch their own children engage in a given task or exploration with an iPad-based application provides them with an eye-opening experience about how their children think and how the digital technology can support the development of their mathematical thinking. I hypothesize that offering teachers a small by powerful set of tools to use in their classrooms will be much more beneficial to them as teachers, as well as to students as learners—this argument has also been advanced by Goldenberg (2000). With a small number of recommended tools, it can be easier to provide teachers with specific information on what problem the tool can help solve so that the tool can be used purposefully. Teachers also need to be aware that certain tools that promote repetitive practice may have good short-term pay-off but can also run the very big risk of compromising students' confidence, curiosity and flexibility.

References

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