Research on Teaching and Learning Mathematics

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Research Partnerships

TRENT UNIVERSITY
TRENT MATHEMATICS EDUCATION RESEARCH COLLABORATIVE

OISE U T
Dr. Eric Jackman Institute of Child Study
UNIVERSITY OF TORONTO
& THE ROBERTSON FOUNDATION

ONTARIO MINISTRY OF EDUCATION

Social Sciences and Humanities Research Council of Canada

Conseil de recherches en sciences humaines du Canada

ONTARIO SCHOOL BOARDS: KPRDSB, SMCDSB, TDSB, TCDSB, WCDSB
INDEPENDENT SCHOOLS: BSS, ICS
Current Research Team 2013-2014

Tara Flynn
Sarah Bennett
Rich McPherson
Zack Hawes

Joan Moss
Bev Caswell
Shelley Yearley
Sarah Naqvi
Some Questions

1. What have you learned about teaching/learning in your research?

2. Professional Learning that has positive effect on student learning?

3. Future research directions?
What have you learned about teaching and learning mathematics through your research program?

Go to: www.tmerc.ca
Study of Mathematics Teaching & Learning

Content research interests: Algebra, Fractions, Spatial Reasoning
Pedagogy research interests: Enriched inquiry-based learning opportunities, Technology use
Teacher Efficacy
The teacher’s belief that he or she can help students learn mathematics

Student Efficacy
The student’s belief that he or she has the ability to learn mathematics
Why Student Efficacy Matters

Students who believe they will be successful
- set higher goals for themselves
- try harder to achieve those goals
- persist through obstacles.

Student efficacy is a predictor of student achievement
Why Teacher Efficacy Matters

High efficacy teachers:

1. Are more likely to try out new teaching ideas that involve risk
2. Use classroom management approaches that stimulate student autonomy and reduce custodial control
3. Attend more closely to the needs of lower ability students
4. Persist, even when faced with challenges

Bandura, Tschannen-Moran, Ross, Bruce
Mastery Experiences
STUDENT

“I learned something challenging! I’m really good at this!”

Dance Competition, Kingston Jamaica (Jamaican Self Help)
"I helped you learn something! I'm really good at teaching!"
Efficacy Explanatory Diagram

Student mastery experience

Greater level of persistence and effort by students

Teacher mastery experience

Increased number of student mastery experiences

Greater effort to help students learn

Bruce & Ross 2008
Ross & Bruce 2007
Which teacher professional learning models seem to be working?
Characteristics of High Impact Professional Learning

- Collaborative
- Classroom-embedded
- Cyclical and sustained
- Constructive
- Committed

Bruce & Flynn, AJER 2013
Bruce, Flynn, Stagg-Peterson, 2011

<table>
<thead>
<tr>
<th>Generally Less Effective</th>
<th>Generally More Effective</th>
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<tbody>
<tr>
<td>Expert or leader directed, with imposed goals and expectations</td>
<td>Collaborative in nature, with shared goals and expectations (alignment and agreement)</td>
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<tr>
<td>Learning environments that are primarily outside the classroom and require translation by educators to the classroom</td>
<td>Classroom-embedded learning where the primary site of inquiry is the classroom context (requires an opening of doors, and shared risk-taking)</td>
</tr>
<tr>
<td>Short, workshop oriented session(s) or meeting(s) that gloss over the mathematics content</td>
<td>Cyclical: Iterative and sustained inquiry that focuses on key mathematics ideas and content</td>
</tr>
<tr>
<td>Deficit orientations toward educators and/or students (&quot;fix-up&quot; models)</td>
<td>Constructive: Asset orientations toward educators and/or students (What can students do? And what more can they learn?)</td>
</tr>
<tr>
<td>Disengagement, mistrust and resistance to evidence are high amongst members of the group</td>
<td>Committed: Trust, ethic of care and accountability are high amongst members of the group</td>
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</table>
The Case of Fractions Collaborative Action Research

Multi-year
Mixed-methods
Large-scale
Why Focus on Fractions?

- Difficult to learn, difficult to teach
- Fractions misconceptions present the most “serious obstacle to the mathematical maturation of children” (Behr, Harel, Post & Lesh)
- Underpins proportional reasoning, spatial reasoning, algebraic reasoning, probability
- Essential to STEAM fields
Fractions can be Fatal...

...Pediatricians, nurses, and pharmacists... were tested for errors resulting from the calculation of drug doses for neonatal intensive care infants... Of the calculation errors identified, 38.5% of pediatricians' errors, 56% of nurses' errors, and 1% of pharmacists' errors would have resulted in administration of 10 times the prescribed dose.

(Grillo, Latif, & Stolte, 2001, p.168)

(Bruce & Ross, 2009)
4 Key Findings

- Match context to representations
- Increase explicit instruction of unit fractions
- Introduce and continually extend the use of number lines (longevity)
- Explore fractions and their use throughout the course/year, rather than a ‘unit of study’
Fraction Situations & Representations

Lucy walks 1 1/2 km to school. Bella walks 1 3/8 km to school. Who walks farther? What picture would help represent this fraction story?

We suspected linear models would be used...
\[
\frac{2}{5} = 0.40 = 0.4 = \text{Fourth} = 40\%
\]

Of the pie is eaten

Bobby had \( \frac{3}{5} \), so it was wasted because instants part of a part meant one.

\[
\frac{4}{10} = \frac{2}{5}
\]

40% of the pie is eaten

\[
\frac{200}{500} = \frac{2000}{5000}
\]

pie

\[
\frac{200000000}{500000000}
\]
But Circles aren’t always helpful...

1. Circles are hard to partition equally (other than halves and fourths, eighths)
2. They don’t fit many situations
3. It can be hard to compare fractional amounts.
Part-whole Set Relationships

Important, but neglected...
Other Part-whole Relationships
Important, but neglected...

Linear models

2D Area models

3D models
(volume; surface area)
Linear Models (eg., for Making Comparisons)

- $2/5$ of the marbles are blue
- $3/10$ of the marbles are mint green
- Are there more blue or mint green marbles in the bag?

(Petit et al., 2010)
Explicit Instruction of Unit Fractions

- Composing and decomposing fractions using unit fractions such as $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{4}$
- Naming the units - counting by unit fractions
  (1 one-fifth, 2 one-fifths, 3 one-fifths, ...)

\[
\frac{6}{5}
\]

'6 one-fifth units'
Fractions on Stacked Number Lines

- $\frac{1}{2}$
- $\frac{1}{3}$
- $\frac{1}{4}$
- $\frac{1}{5}$
- $\frac{1}{6}$
- $\frac{1}{8}$

Found on the corner of desks in some classrooms in Japan.
Increase
No change
Decline

Sample data from fractions research, where students worked with fractions throughout the year instead of as a stand alone “unit” of study.

Bruce, Flynn, Yearley 2012
Understanding Fractions

Understanding fractions is a fundamental concept in mathematics, as it involves recognizing the parts of a whole. Fractions are used in various aspects of daily life, such as cooking, budgeting, and measuring. In the context of learning, understanding fractions is crucial for developing critical thinking and problem-solving skills.

When teaching fractions, it is important to use a variety of teaching methods to cater to different learning styles. Visual aids, such as fraction bars or pie charts, can help students visualize the concept of fractions. Additionally, using real-life examples can make the concept more relatable and engaging for students.

Common misconceptions about fractions include:
- Assuming that fractions are only used in everyday life, while in reality, they are used in many scientific and technical fields.
- Thinking that fractions are only used for cooking, but they are used in many other fields, such as engineering and economics.

To overcome these misconceptions, it is important to provide students with a variety of opportunities to practice and apply their knowledge of fractions in different contexts.

References:
- The use of visual aids and real-life examples can make the concept of fractions more accessible and engaging for students.
- Understanding fractions is essential for success in higher-level mathematics and science courses.

For more information, please visit www.mathgains.ca.
- Is it like multiplication? Like 2 groups of 6?
- Or is it like addition, like 2 + 5?
- Is it like division, 2 divided by 5?
- Two, fifths might equal 1 tenth

St 2: So we know that we've covered those ones in.
Exploring Unit Fractions

1-A Make reasonable estimates using proportional reasoning

1-B Demonstrate equal partitioning using area, linear, and set models

1-C Compose and decompose mixed and proper fractions using models, with unit fractions as the base

1-D Connect composing and decomposing using models to symbols

1-E Compose and decompose mixed and proper fractions using models and symbols

2-B Recognize and generate simple equivalent fractions using area, linear, and set models

2-C Estimate and compare mixed and/or proper fractions using models and referring to benchmarks

2-D Compare fractions with different numerators and/or different denominators of symbolic representations

2-E Connect merging and splitting of models to symbols

2-F Find equivalent fractions symbolically to order and compare quantities

3-A Add and subtract fractions with like denominators using models

3-B Add and subtract fractions with like denominators using both models and symbols

3-C Add and subtract fractions with unlike denominators using both models and symbols

3-D Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models

3-E Connect models to symbols

3-F Add and subtract fractions with unlike denominators using both models and symbols

Addition and subtraction operations with fractions

Equivalence & comparing fractions

Created by Dr. Cathy Bruce, Tara Flynn and Shelley Yearley
Inspired by Dr. Jere Confrey's work
Based on international research and Ontario classroom research
Informed by feedback from classroom teachers and student thinking
Funded by Ministry of Education, Ontario
So...

Does collaborative PL work?

If it authentically enacts the characteristics of high quality PL
Grade 9 Applied EQAO Math Scores: Last 5 years

Grade 9 Academic
School A: 78-92%
Province: 77-84%
School B

Grade 9 Applied EQAO Math Scores: Last 5 years

Grade 9 Academic
School B: 88-88%
Province: 77-84%
What is happening at school A?
Interviewed the Grade 9 teachers of school A and the Grade 7 & 8 teachers of feeder schools

**PD Experiences**
- 5 years of collaboration
- Self-directed and teacher initiated learning goals from Grades 7-9
- Supported by principals
- Grade 7-9 teachers meet regularly to develop continua of learning (lesson trajectories)
- Patterning and algebra PD in larger group and hub (2013)
- Grade 8 teachers go to the secondary school and participate in grade 9 academic class and grade 9 applied class
- Grade 9 teachers come to the grade 8 classes and teach 2 lessons in the feeder schools

**Over-arching Goal:**
"Set Kids Up for Success"
Culture of collaboration (planning, communication, co-teaching, mirroring: “It is expected: we collaborate to ensure student success”)

Positive attitudes about academic and applied learning (with ‘accurate’ streaming and case by case analysis)

Consistency (same team of 7-9 teachers for 5 years)

Invested and PASSIONATE for this age group AND FOR MATH

Cyclical approach to the curriculum (bi-weekly reviews and circling back through strands)

Professional pride and accountability to one another and the students

Customized, personalized, and focused instruction
Future Research Questions?

Is the draft fractions learning pathway accurate for some/many/most learners?
(current partnership project with Ministry)

How does spatial reasoning equip children for mathematics learning across strands? What are the benefits of a specialized curriculum? Does spatial reasoning predict geometry and other mathematics such as number?
(searching for funding)

What more can we learn about the capacity of young children in mathematics?

Stories of resistance
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Trent University
A Few Parting Thoughts
Countries performing better than or as well as Canada — Mathematics

### Better than Canada*

2012

Mathematics - Overall

Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Korea, Macao-China, Japan, Liechtenstein, Switzerland

### As well as Canada*

The Netherlands, Estonia, Finland, Poland, Belgium, Germany, Vietnam

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PISA data that may be in the PUBLIC EYE
<table>
<thead>
<tr>
<th>Level</th>
<th>Lower score limit</th>
<th>Percentage of students able to perform tasks at this level or above</th>
<th>Characteristics of tasks</th>
</tr>
</thead>
</table>
| 6     | 669.30            | 3.3% of students across the OECD and 4.3% in Canada           | Students at Level 6 of the PISA mathematics assessment are able to successfully complete the most difficult PISA items. At Level 6, students can:  
- conceptualize, generalize and use information based on their investigations and modeling of complex problem situations, use their knowledge in relatively non-standard contexts.  
- link different information sources and representations and move flexibly among them.  
- demonstrate advanced mathematical thinking and reasoning and apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations.  
- reflect on their actions, and formulate and precisely communicate their actions and reflections regarding their findings, interpretations and arguments, as well as explain why they were applied to the original situation. |

Complex problem solving L6 (4.3%)  
2.4% at level 6 in 2009

1     | 357.77            | 92.0% of student across the OECD and 96.4% in Canada           | At Level 1, students can:  
- answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined.  
- identify information and carry out routine procedures according to direct instructions in explicit situations.  
- perform actions that are almost always obvious and follow immediately from the given stimuli. |

Basic Procedures L1 (96.4%)
Procedures and Problems
Go Hand-in-Hand

What is the perimeter of this figure?

Your dog Cody needs a pen. You have 52 metres of fencing.

Design three possible pens for Cody using all the fencing.

Resist the temptation to artificially separate out concepts and procedures, skills and problems, knowing and applying...
EQUITY
Across Canada, despite high immigration and income disparities, students succeed

GENDER GAP
Across Canada, the gender gap in favour of boys has remained remarkably stable over the past nine years (11 pts in 2003, 10 pts in 2012)

HAPPINESS & ENGAGEMENT
Students in Canada report being happy at school in contrast to other countries...

TECHNOLOGY-BASED MATH COMPONENT OF PISA
CANADA RANKED 8