KNOWLEDGE BUILDING IN MATH

KNOWLEDGE BUILDING IN MATH: HOW DO WE HELP STUDENTS TO THINK LIKE MATHEMATICIANS?

How can we help support the shift from "learning math" to students' "thinking like mathematicians?" Like all Knowledge Building, the approach in math is collaborative and inclusive; it requires that students and teachers work together on achievement objectives, and that students take high-level responsibility for goals and outcomes — and especially for idea improvement. In this section, we focus on how Knowledge Building can be applied to mathematics across all Grade levels.

CURRICULUM CONNECTIONS

A KB approach to math engages students in the **key mathematical processes** that support effective learning in math, as outlined in the Ontario curriculum document. When students are engaged in Knowledge Building to advance mathematical understanding, they are:

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

> "Problem solving is central to learning mathematics. By learning to solve problems and by learning through problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction" (*The Ontario Curriculum Grades 1-8, Mathematics, 2005,* p. 11).

Moreover, Knowledge Building promotes the development of an effective pedagogical system for math learning (Anthony & Walshaw, 2007), which includes:

- A non-threatening classroom environment
- Worthwhile math tasks
- Classroom discourse
- Tools and representations

The mathematical processes and the elements of an effective pedagogical system outlined above are underscored in the Vision for the Mathematical Learner, which informs Ontario's Renewed Math Strategy, 2016 (see http://www.edu.gov.on.ca/eng/policyfunding/memos/april2016/min_math_strategy.html).

The Vision for the Mathematical Learner is summarized in the chart.

Vision for the Mathematical Learner	
Understands Math Concepts	Students engage in learning opportunities that: • Develop conceptual understanding • Make connections among mathematical ideas • Show mathematical thinking using models, tools, and representations
Is Proficient with Facts, Skills, Procedures	Students engage in learning opportunities that: • Provide meaningful practice in a variety of ways • Foster the use of appropriate mathematical language, notations, and symbols to communicate • Support consolidation and mastery of learning
Engages in Mathematical Processes	Students engage in: • Problem Solving • Reasoning and Proving • Reflecting • Selecting Tools and Computational Strategies • Connecting • Representing • Communicating
Demonstrates Autonomy and Self-Regulated Learning	Students have an active role in: • Developing understanding of learning goals and success criteria • Applying criteria, monitoring progress, reflecting, and setting individual learning goals • Developing and practising peer and self-assessment
Displays A Positive Attitude Towards Mathematics	Students experience a learning environment that: • Feels safe to wonder and take intellectual risks • Develops self-efficacy, resilience, and a growth mindset • Positions them as mathematical thinkers and doers • Fosters an appreciation for math as an important tool in daily living

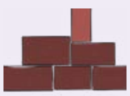
CONNECTIONS TO THE KNOWLEDGE BUILDING PRINCIPLES

The key math strategies described above are aligned with the **12 Knowledge Building Principles** (see pgs. 20-26), which support student engagement, confidence, and achievement in math. By providing explicit opportunities for students to work with the KB Principles and to experience what they look and feel like in their math learning, we allow all students the opportunity to develop the skills of a mathematician. For example:

- **Real Ideas, Authentic Problems:** This principle involves engaging students in problems involving math that spark their natural curiosity, which can help them both develop their understanding and form positive attitudes towards the subject (Colgan, 2014).
- **Improvable Ideas:** This principle centres on the notion that "all ideas are improvable" and helps to encourage a "growth mindset" (Dweck, 2006), which, when applied to the domain of math, can help boost learner confidence and development (Sirois, 2014).
- **Knowledge Building Discourse:** The importance of mathematical discourse — "math talk" — to achievement and engagement in the subject is well known (NCTM, 2000; National Research Council, 2001). In a Knowledge Building community, opportunities for members to engage in peer-to-peer Knowledge Building Discourse are a priority.

The chart below suggests an alignment between the KB principles, and different aspects of the Vision for the Mathematical Learner. It includes examples of student KB Discourse from a Grade 6 KB classroom that illustrates engagement for each category.

KB Principle	Vision for the Mathematical Learner	Student KB Discourse
Real Ideas, Authentic Problems	Displays a Positive Attitude Towards Mathematics	"What percentage of the world has access to clean water?" "How do some people live on $1 a day?" "Why might the amount of this shopping list for our pie ingredients be over the budget we had?"
Idea Diversity	Understanding Math Concepts Is Proficient with Facts, Skills and Procedures Engages in Mathematical Processes	In a KB Circle: Student 1 Shared: "13% of Afghanistan's population has clean water." Student 2 shared: "84% of Cambodia's population does not have clean water." "I never realized that there are two ways to represent similar information." "There is only a 3% difference between the two countries."
Improvable Ideas	Understands Math Concepts Is Proficient in Facts, Skills, and Procedures Engages in Mathematical Processes Displays a Positive Attitude Towards Mathematics	Student 3 shared that 13% of Afghanistan's population actually does not have clean water; the class then decided to use the stat from Cambodia to test this idea: "Let's draw this idea out in a circle graph/pie chart, and see if it makes sense." They discovered it was a misconception: "If 13% of the population doesn't have clean water then the country would not be in a Water Stress."
Knowledge Building Discourse	Understands Math Concepts Engages in Mathematical Processes Displays a Positive Attitude Towards Mathematics	Students were working on designing the ideal city. They encountered a couple of problems: "If we are saying that all tuition will be paid for homeless people in our Helping Hands Shelter, where would the money come from?" "How do we handle population growth if we only have 5 residential homes in this city?" "Building onto the theory of dividing by 5, I'm thinking we could use the calculator to compare solutions."
Community Knowledge, Collective Responsibility	Understands Math Concepts Is Proficient with Facts, Skills and Procedures Engages in Mathematical Processes Demonstrates Autonomy and Self-Regulated Learning	Students were working on a math problem involving distributing pies: "We need everyone's idea of how to multiply these measurements enough times so that there is enough pie crust for all of the pies." "If we don't figure this out, we will run out of ingredients, and/or everyone won't have a pie to make."
Epistemic Agency	Understands Math Concepts Engages in Mathematical Processes Demonstrates Autonomy and Self-Regulated Learning	"I'm skeptical of your theory. If we used that strategy, this would change the meaning of the number. I think we should take a closer look at place value here."

KB Principle	Vision for the Mathematical Learner	Student KB Discourse
 **Rise-Above**	Understands Math Concepts Is Proficient in Facts, Skills, and Procedures Engages in Mathematical Processes Demonstrates Autonomy and Self-Regulated Learning	“We are missing something because all of our ideas for population numbers don’t make sense.” “We are going back and forth with too many numbers 15,000-100,000. What are we missing?” “What do we need?”
 **Embedded, Concurrent, & Transformative Assessment**	Engaging in Mathematical Processes Demonstrates Autonomy and Self-Regulated Learning	Accountable Number Talk/KB Circles: “I’m wondering why my answer is different than everyone else’s. Did anyone notice that?” “I see you went wrong, you just needed to subtract the other group of 8.” “Let’s try to use more of the math terms from the word cloud, to increase the number of mathematical terms in our KF discussion” (see pg. *84-88* for more information on the word cloud).
 **Pervasive Knowledge Building**	Displays a Positive Attitude Towards Mathematics	“This weekend, my mom was sharing about how Parent Council takes a certain percentage of pizza sales and that money helps to pay for some school trips. Would something similar work for our baseball ticket sales and school fees?”
 **Constructive Use of Authoritative Sources**	Engaging in Mathematical Processes Demonstrates Autonomy and Self-Regulated Learning Displays a Positive Attitude Towards Mathematics	“How do mathematicians or experts use mathematical terms, when speaking about math?” “Let’s compare our terms with expert terms.” “We used 5 expert terms…” (see pg. *84-88* for more information). “How would a real city planner be able to estimate the population when building homes in the community?”
 **Democratizing Knowledge**	Engaging in Mathematical Processes Displays a Positive Attitude Towards Mathematics	“Let’s ask the whole group if they have any ideas of where the buildings should be located, since we are making changes from the 2D map to our 3D model.” “____ inspired me to think of this solution in a different way.”
 **Symmetric Knowledge Advance**	Demonstrates Autonomy and Self-Regulated Learning Displays a Positive Attitude Towards Mathematics	“I’m wondering if we could take the baseball ticket money, and give it to the Helping Hand University Fund.” “Can everyone try to explore my idea? I think it may work with everyone’s help.”

> “The 12 Principles are components of a complex process, and the really good news is that any single one that you unlock helps to unlock the others…focus on whichever one appeals to you.” — *Marlene Scardamalia*

BUILDING THE FOUNDATIONS OF A KB COMMUNITY IN THE MATH CLASSROOM

A safe classroom environment and a collective culture that values deep thinking, wonderment, and idea improvement are key to a healthy and thriving Knowledge Building community. The importance of nurturing such classroom conditions cannot be overstated! If learners don't feel safe, if there isn't a commitment to idea improvement and risk-taking, it will be a great challenge to build a healthy Knowledge Building community. Indeed, creating the kind of climate and culture that allows KB to thrive in the classroom can start from the first day of school and can encompass many different elements of the classroom (see the section Growing a Knowledge Building Culture, pgs. *18-33*). Engaging in idea improvement around mathematics offers some great opportunities for concurrently both developing a KB culture and for engaging students in deep thinking and key mathematical processes.

SOME KEY IDEAS

There are a number of approaches teachers can take to engage Knowledge Building for math. For example, students can tackle a pre-defined math problem, or they can grapple with a real-life issue where math can be emphasized. Whatever the approach, there are some common key ideas and practices teachers can adopt to engage critical Knowledge Building Principles in their classroom:

- **FOCUS ON COLLABORATIVE *KB DISCOURSE*:** Collaborative strategies such as Gallery Walks that allow for *Idea Diversity,* and *Democratizing Knowledge* can be integrated to help students build on one another's ideas and strategies. To ensure that all ideas become a part of the community's collective knowledge, after small group work or activities like Gallery Walks, ideas from all students are discussed by the whole class in **KB Circles**.
- **MAKE THE GOAL OF DISCUSSION *IDEA IMPROVEMENT*:** Collaborative discourse focuses on idea improvement rather than getting at the right answer speedily. The discussion is a key time for the teacher to assess students' level of understanding and to plan the next instructional move.
- **INTEGRATE *CONSTRUCTIVE USE OF AUTHORITATIVE SOURCES*:** Authoritative sources that engage whatever concept is being dealt with, and/or results of student experimentation and testing (e.g., models and drawings) would be used during discussions. Any student artifacts or resources created during small group or individual work time could serve as objects of inquiry for discussion. (Alternately, these artifacts could also be photographed and put inside **Knowledge Forum** as objects to ground further discussion and exploration (see pgs. *96-108* for more information).
- **INTEGRATE MINI-LESSONS FOR SUPPORTING PROCEDURAL KNOWLEDGE:** Teacher-directed mini-lessons can be one approach for supporting learning of mathematical facts, vocabulary, conventions, etc., such as times tables. These should be integrated within collaborative activities as teachers assess gaps in student understanding.

The following section outlines a tangible process that focuses on bringing important KB principles, such as ***KB Discourse***, and ***Community Knowledge, Collective Responsibility*** to life through practical strategies and concrete activities in the mathematics classroom. (Watch the video series Innovations in Thinking and Learning: Reflections and Lessons - Grade 6 here or Math in Action here to see KB mathematics classes in action; or, check out the KB Case Studies package at http://thelearningexchange.ca/wp-content/uploads/2017/04/Knowledge-Building-All-Case-Studies-Accessible.pdf for a variety of math-related case studies).

POSSIBLE METHOD:

This example is modelled on the work of Hamilton Wentworth teacher Suzana Milinovich. A case study detailing her classroom practice is also provided in this resource beginning on pg. *84*.

- The teacher focuses on a fostering an effective math strategy, for example fostering collaborative classroom discourse around mathematics.

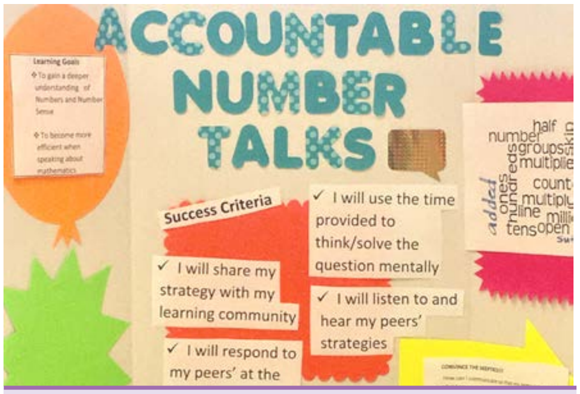
KB DISCOURSE

- The teacher selects a strategy to encourage communication in the math classroom, for example Number Talks. Other strategies include Gallery Walks, Math Congress, Bansho, etc.).
- The teacher creates a physical space optimal for collaborative whole-class discourse through **Knowledge Building Circles**. For example, in this case, a community table made of a whiteboard was placed on top of rectangular tables in the centre of the room as a space dedicated to collaborative discourse.
- Teachers and students co-design the learning goals and success criteria for Accountable Number Talks to help lay out how classroom discourse will be organized and conducted.

Community Knowledge Collective Responsibility

Learning Goals	Success Criteria
To gain deeper understanding of numbers and number sense	I will share my strategy with my learning community
To become more efficient when speaking about mathematics	I will respond to my peers at the right time, using scaffolds wherever possible
	I will use the time provided to think/solve the question mathematically
	I will listen to and hear my peers' strategies

- The teacher deliberately explains one of the learning goals: "To become more efficient when speaking about mathematics." She begins by introducing one **KB math scaffold** (sentence stem) — *"I agree with because/I disagree with because."* The teacher models how to use the scaffold and communicates the importance of using scaffolds when speaking about mathematics. This first scaffold is written down on paper and is laid out in the middle of the table so that it is made visible to the students.



Accountable Number Talk Success Criteria and Learning Goals

- Students collaboratively tackle a math problem by gathering together in a circle around the table. The teacher introduces a math problem and facilitates the discourse (see *"What does a collaborative problem solving session look like?" on the next page to explore in more detail how these discussions can unfold*). The aim is to work towards the gradual release of responsibility for facilitation of the conversation to the students themselves.

- The KB community is at the preliminary stage of discourse as represented by only two scaffolds on the table: *"I agree/disagree with ________'s solution because…"* (see Photo 4). This scaffold helps students to learn to share diverse ideas and to articulate their math thinking. It also helps them to disagree respectfully and to work with ideas that both support and contrast their own — both key capacities for creative work with knowledge. As a next step, students can build off this scaffold and start to frame their ideas and pose questions using key KB Scaffolds "My theory is" or "I still need to understand". "This growth in scaffold use supports the goal and ultimate purpose of Knowledge Building Discourse, which is to move beyond *argumentation* and *idea-sharing* towards actively developing theories, building upon ideas and *creating new knowledge*. Ultimately, KB Discourse is concerned with the questions: ***How can we make this better? Are we headed in a promising direction? Are we getting to the heart of the problem?***

- Over time, the teacher can both introduce new scaffolds to students, and also listen for scaffolds emerging from students' naturally-occurring dialogue to deepen the discourse (e.g., "*Building onto ______'s idea…*"). The teacher records the new scaffolds and adds them to the table (see Photo 5). At the next Accountable Number Talk/KB Circle, the teacher reintroduces the new scaffold to the students.

- Every day the class reflects on the scaffolds (part of the Accountable Talk success criteria) and the newly added scaffolds.

- After each Accountable Number Talk/KB Circle, students individually reflect on the use of a math scaffold in their Math Journals (e.g., setting a goal to use a specific math scaffold next time, how one was used and how it helped the learning/understanding of the knowledge community, etc.). Students also reflect on and record the mathematical thinking that they shared during the discussion, or the ideas that a peer contributed, in pictures, numbers and/or words.

The **KB Scaffolds** (see pgs. *42-44*) represent the kind of talk that helps students actively develop theories, to go beyond opinion and information-sharing to growing ideas, and constructing new knowledge. KB scaffolds represent very powerful epistemic markers — essentially, ways of thinking and knowing — that can help support student engagement in expert-like processes and behaviours, and help grow a culture of knowledge creation within a classroom.

Photo 4. The classroom table where KB Circle / Number Talks take place. At the onset, the table has only two scaffolds.

Photo 5. The addition of scaffolds around the table occurs over time as discourse deepens.

WHAT DOES A COLLABORATIVE PROBLEM SOLVING SESSION LOOK LIKE?

This section provides an elaboration on the work described above. More specifically, we narrow down on what a KB approach to tackling an assigned math problem can look like in the context of community Knowledge Building. The focus here is on outlining some practical steps and important teacher moves that take place during the process of collaborative problem solving.

POSSIBLE METHOD OF DELIVERY

- **A key mathematical strategy that helps bring to life key KB Principles has been selected** (e.g., fostering collaborative KB Discourse in the math classroom) and a plan for implementation has been set (as outlined on the previous page).
- **Teacher introduces a provocation or problem** related to the Big Ideas in the Mathematics curriculum to ignite wondering and questioning. For instance, the question: "What is 20% of a number"? (Overall Expectation #1 in Gr. 5 Number Sense and Numeration).
- **Students are given sufficient independent think time** (approximately 5 minutes) to puzzle over the problem individually (e.g., mental, visible, non-permanent flexible think space).
- **1st KB Circle/Accountable Number Talk:** As a whole group, students are given the opportunity to share their initial thoughts, understandings, misconceptions, and any need for clarifications on the problem (5-10 minutes).

Symmetric Knowledge Advancement

- **Investigations:** Students break out independently or in pairs to work on skills and engage in mathematical processes such as reasoning, problem-solving, theory-building and communicating their mathematical thinking.
- **2nd KB Circle/Accountable Number Talk:** Students come back as a whole group to share their theories, solutions, justifications, reasoning, etc., through the use of KB scaffolds.
- **Reflection/Exit Cards:** Students go back to complete their individual Math Journal entries in which they reflect on their own use of scaffolds and the ideas they discussed, or they can reflect on the ideas others' contributed.
- **Assessment is ongoing:** The teacher is uncovering the misconceptions and identifying specific learning needs of individual students and the whole group to inform and guide the teachers' next instructional move.
- **Strategize about the next instructional move:** In which direction do I need to go?

  IF most students have deep or recurring misconceptions, THEN → **Direct teaching to the whole group** (e.g., deliver a mini-lesson to the whole class).

- *IF most students have a good grasp of the concepts, THEN* → **Conduct guided instruction** (e.g., conduct a small group mini-lesson; for instance, take a small group, have a manipulative at the table, and show them concrete examples of the problem).

  IF the community grasps the concepts, THEN → **Support further theory development with the class** (plan the next Accountable Number Talk/KB Circle problem to build on the initial problem and to help push thinking further). The teacher reflects and makes links to curriculum expectations from students' theories. For instance, students are understanding fractions and decimals but are not making the connection to money. After a certain period of time, the teacher decides it's time to bring forth this idea to students in order to help students move thinking forward.

This process can be continued and repeated regularly.

INTEGRATING TECHNOLOGY:

Knowledge Forum (KF) can be introduced to students in this process to help make students' thinking visible to the whole community, and to provide another safe space for all students to contribute their ideas. The open, flexible discussion platform and the program's multimedia features allow for differentiated instruction in order to meet the needs of all learners (see pgs. *96-108* to learn more about KF).

Figure 7 depicts a Knowledge Forum "view." The red and blue notes represent student notes. Figure 8 shows a note when opened up. In this view, students are working on their "Decimal Theories". Images taken from classroom work have been uploaded onto the background of the view to support discussion.

Figure 7. A KF "view" on decimal theories. Student notes are indicated in blue and red squares, with images take from classroom work along the bottom.

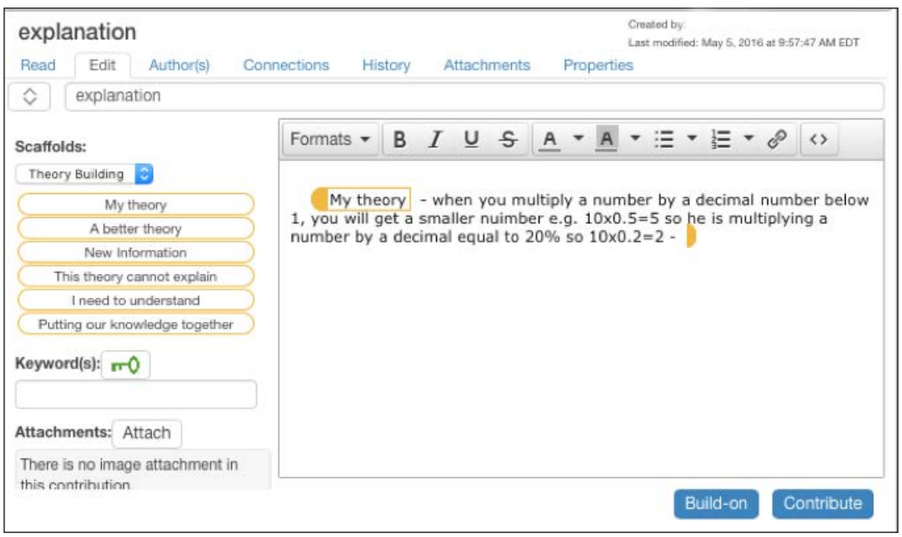


Figure 8. An open KF note. The use of the scaffold support "My theory" used here and framed in yellow, helps to structure student thinking and writing.

WORKING WITH REAL-LIFE ISSUES AND CROSS-CURRICULAR QUESTIONS WHERE MATH CAN BE EMPHASIZED

It can be a challenge to find authentic math problems — things the students actually wonder about or need to deal with in real life. In the natural sciences, for example, students may wonder about many things; **how can teachers help them see the mathematical side of what they wonder about?** For instance, information about life spans of different animal species is sure to raise questions — why do some animals live only weeks, while others live more than a hundred years? Students will generate theories such as 'larger animals live longer.' From these notions, *Real Ideas, Authentic Problems* arise as students test their theories, discover that some facts do not fit, and then try to revise and improve their ideas. **Mathematics can play a part in all of this and can be brought to the fore through explicit integration of math strands within broader work.**

In what follows, we outline two examples of how mathematics was brought to the fore as one major component of cross-curricular work as part of Suzana Milinovich's evolving KB practices in her Grade 6/7 classroom.

EXAMPLE 1: EQUA-CITY!

A Social Studies provocation inspired the Big Question: "*What is Utopia*"? Building off many areas of study, students began to envision and hope for a future in which equality and inclusiveness would be very visible.

Students work on building their Equa-City by engaging in active research, design and collaboration.

- The students wanted to create their own ideal city, and set to work to design a futuristic city they named Equa-City. They proposed ideas and then created models and individual structures to contribute to Equa-City. Students' structures had to fit together as a whole, so they had to work collaboratively to make their city plan work. Several KB Circles and investigations took place over the course of this work to help students build off one another's ideas and work out any issues or problems that were arising in the design and construction of Equa-City.
- Authoritative sources and experts were consulted to help the students plan their city. For example, an artist from the Ontario Arts Council came in to help students design and construct their buildings. After students created their initial city design, a city planner visited the classroom to help students improve their ideas. For instance, discussion with the expert planner helped students realize they did not account for grass and green space, or that they were missing some important public buildings and services, such as a hospital.

- Students also asked the city representative challenging questions such as, “How do you plan for a population that doesn’t exist yet?” Students also uncovered technical problems with their structures, such as discovering that some structures were too tall for their respective locations and had to be rethought and relocated.
- The teacher also involved the students in making cross-curricular connections between mathematics and the other subject areas they were engaging. The Sketchnote graphic (right) represents the teacher’s reflection of the Knowledge Building occurring in her classroom community, and the cross-curricular links that were being made throughout the process.

A Sketchnote graphic depicting cross-curricular links and connections

Highlighting the mathematical side of building the “Equa-City”:

When a vibrant Knowledge Building community is built, **Pervasive Knowledge Building** naturally occurs. In this case, students were very invested in designing and redesigning their city and thinking about social justice and equity. An integral and continual part of this process was working with mathematical ideas and concepts.

- Classroom discussions engaged topics like **population numbers** and **data management**, which have direct links to mathematical concepts.
- The teacher recognized that there were many opportunities to engage students in mathematical thinking and develop critical math skills (reasoning, problem-solving, communicating) within the context of this authentic inquiry work.
- Students engaged in mathematical discourse around important math concepts during KB Circles and group discussion. For example, part of the process of building the city’s structures was translating students’ plans from 2D to 3D. KB Circles were used to help the group decide how to undertake this process successfully and bring their visuals to life. Students engaged important mathematical ideas in **geometry, measurement,** and **spatial reasoning.** Moreover, students wanted to ensure equity and education for all in their city, so they devised a plan to take a percentage of baseball ticket sales to fund public education. They worked on **data management issues**, and problems involving **percentages and fractions** as they designed both the architectural, social, and cultural aspects of their city.

Students build 3D models of their designs and structures.

EXAMPLE 2: ENHANCING OUR COMMUNITY!

This example features the work of Waterloo District School Board teacher Rhonda Hergott. The text below is adapted from her blog *Thoughts from the Behind the Desk*. The images below can be found there, as well as links to student work and videos. Access Rhonda's blog here: https://rhondahergott.wordpress.com

- Students were provoked into learning more about their own small community when the question was posed to them: *"Is our community in need of a mini-golf course?"*
- Students contacted StatsCan to collect demographic information about their community, and discovered how difficult it was to understand and interpret raw data. Students agreed that they needed more information to work from.
- Students decided what information they needed to acquire, and collaboratively designed their own survey. Students also discussed strategies for how they would deliver the survey and decided on methods to access a random sample of community members.

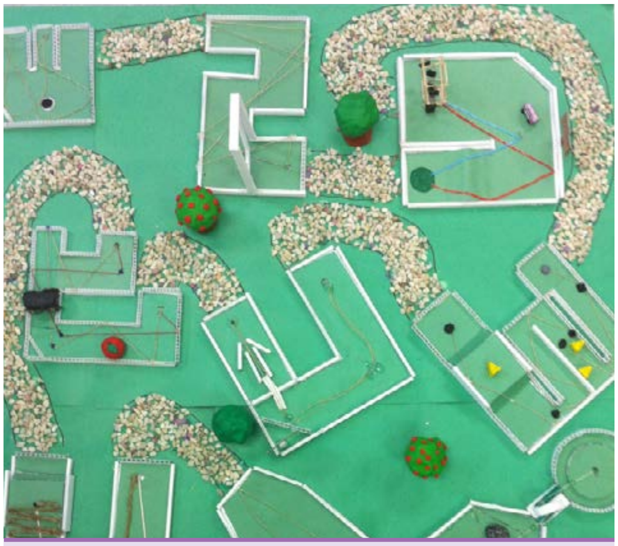
> "**There is no real situation where we would only use one concept in math to solve a problem.** Yet in education it seems that is the only way we teach it. Math concepts, independent of one another, disjointed with no purpose other than to complete the 7 questions on page 126 of the textbook. This year my math class is not going to focus on one strand at a time but instead focus on designing, developing and creating a miniature golf course that will represent the small community we live in. The math that will be required to solve this problem will come from all strands and will have a purpose." — *Rhonda Hergott*

- Infographics to communicate findings was chosen as a useful approach for the project. Students analyzed infographics, created success criteria for their own designs, explored various infographic programs, designed drafts and engaged in peer assessment to improve designs.
- The location of the mini-golf course also had to be determined. Students discussed different options and settled on three possible locations. They then headed out to examine each setting to select the most appropriate option. Students used trundle wheels and measuring tools to assess scale and space and also took into account factors such as lighting and fencing.

Students observe, take measurements, and notes during a field trip to a real mini-golf course.

- Students designed mini-golf holes individually. They then undertook field research to learn more about how golf holes are designed, so they organized course. Students also requested and arranged for a local landscape artist to attend the trip and help students plan their own holes, and think about important elements such as accessibility for mobility devices and vehicles.

- Once the mini-golf course was designed, students self-organized into various teams to prepare a presentation for the Apple Butter and Cheese Festival, which, if successful, would result in the class receiving funds to build a transportable version of their golf course. There was a Presentation Team who would create and present the final presentation, a Data Team who would summarize all the data collected and interpreted throughout the project, a Research Team who would determine materials and prices for building a model, a Model Team who would construct a 3D model golf course, and finally, a Documentary Team that would record the whole process (see their movie here!)



A 3D model of the student-designed mini-golf course

Highlighting the mathematical side of designing the mini-golf course:

- Students' work with census data provided wonderful teaching opportunities to engage students in discussions about what a population is in **data management**.

- The teacher also capitalized on the infographics work to have students investigate more closely how images and **numerical representations** help audiences to understand and interpret the information.

- Designing original surveys opened up teaching opportunities for exploring what a **random sample** is and how to obtain one, as well as discussing what the characteristics of a survey are and what makes for a good design.

- The location assessments engaged students in meaningful learning opportunities around **measurement, scale,** and **space.**

- On the mini-golf course they visited during a field trip, students also explored the **experimental probability** of each hole. They all played one round and kept track of their score. The score cards were then used as data for classroom investigation about whether there is a connection between experimental probability and the par for each hole.

The following case study provides an elaboration of what a KB approach to collaborative problem solving can look like in action. Note the use of **KB Discourse, KB Scaffolds,** and **Knowledge Forum** to support the building and advancement of knowledge. (Watch the video series Innovations in Thinking and Learning: Reflections and Lessons - Grade 6 here or Math in Action here; or, check out the KB Case Studies package at http://thelearningexchange.ca/wp-content/uploads/2017/04/Knowledge-Building-All-Case-Studies-Accessible.pdf for this and more math-related case studies).

CASE STUDY (GRADE 6/7): GETTING KNOWLEDGE BUILDING STARTED IN MATHEMATICS

This case study describes Suzana Milinovich's Grade 6/7 class in Hamilton, Ontario. Her class consisted of 25 students, 16 boys and 9 girls. There were 5 children with Special Needs, 3 of whom were working on an Individualized Education Plan with modified expectations in Mathematics. In Suzana's classroom, it was a significant challenge to ignite mathematical conversations among a diverse set of learners in order to advance an entire group's knowledge. Suzana decided to narrow in on setting the conditions for **Knowledge Building Discourse** that would lead to **Democratizing Knowledge** within the group. It was critical for students to understand that all learners in the classroom are relevant contributors to the group's learning, and to have the opportunity to experience this.

KB PROVOCATION:

The majority of students (88%) commenced the year with a negative attitude towards mathematics; an informal survey Suzana asked students to fill out at the onset of the year showed that 85% of the students felt that they exhibited no strengths in the entire subject area. Suzana identified this as her problem of practice and began to embark on a professional inquiry that would shift the attitude and learning of mathematics with these students.

> **What was your greatest challenge?** Students had deep rooted opinions and beliefs surrounding their individual place in a mathematics classroom…They viewed mathematics as a subject in which they either excelled or struggled … Statements such as, "I can't do this" and, "This is too hard" echoed in the room daily. A student even dressed up as an "I hate Math" calculator for our Halloween festivities!

STRATEGIES FOR SUSTAINING IDEA IMPROVEMENT:

1. Number Talks!

With the understanding that student discourse is a highly effective principle in learning, Suzana looked for ways to encourage this in her math class. She provided opportunities for students to learn how to effectively collaborate by having better conversations where all students could be accountable. During mathematics, she introduced **Number Talks** (3 to 5 times per week) in which students, without the teacher, were encouraged to share their individual strategies when solving a number problem.

THE PURPOSE OF NUMBER TALKS WAS TO DEEPEN CONCEPTUAL KNOWLEDGE OF NUMBERS AND NUMBER SENSE, AND TO INTENTIONALLY CREATE A KNOWLEDGE BUILDING CULTURE BY TEACHING EFFECTIVE KNOWLEDGE BUILDING DISCOURSE IN MATHEMATICS.

When sharing during Number Talks, students were encouraged to explain their thinking as though they were trying to convince a room full of skeptics. As students discussed, Suzana simply sketched/recorded responses. **Simultaneously, students were encouraged, as listeners, to become skeptics of their peers' strategy and respectfully question one another's ideas in order to build critical thinking skills.** Suzana would pose questions such as:

- "Can someone re-phrase ____'s strategy?"
- "Can anyone build on ____'s theory?"
- "Would anyone like to ask ____ a question about their thinking?"

Other Important Elements of Number Talks:

- **Learning Goals and Success Criteria** were clearly communicated, re-visited and articulated at the start of each Number Talk Session (see Photo 6).
- **KB Scaffolds:** All Knowledge Builders (students, teacher) model/use KB scaffolds during each discussion.
- **Reflection and Self-Assessment:** Students were encouraged to reflect on the key KB Scaffolds that they used and were comfortable with. Students reflected on the use of a scaffold in their math journals (e.g., they set a goal to use a specific scaffold next time; explained how they used a scaffold and how it helped the learning/understanding of the knowledge community; etc.)

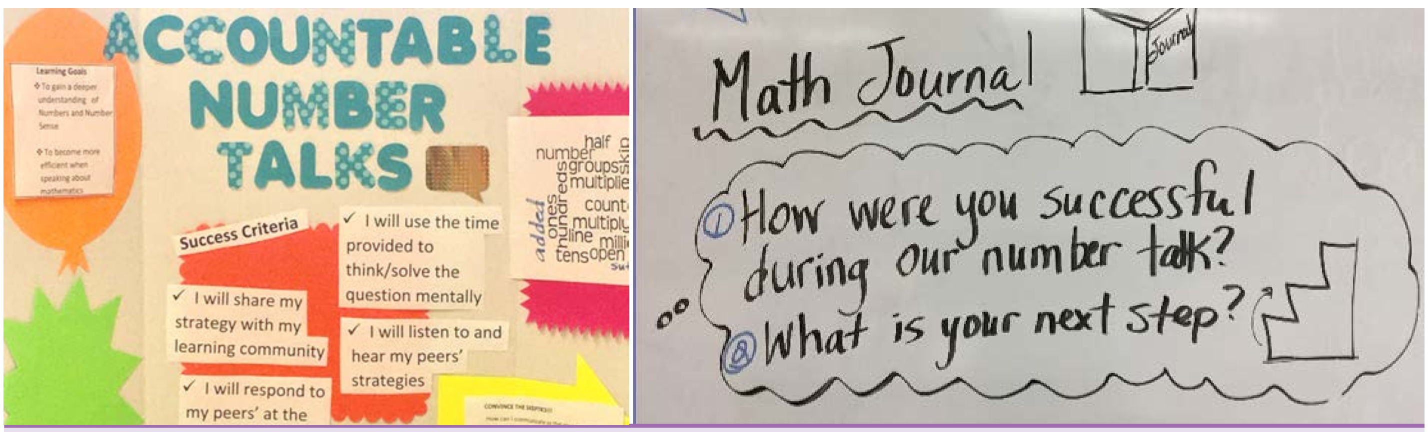
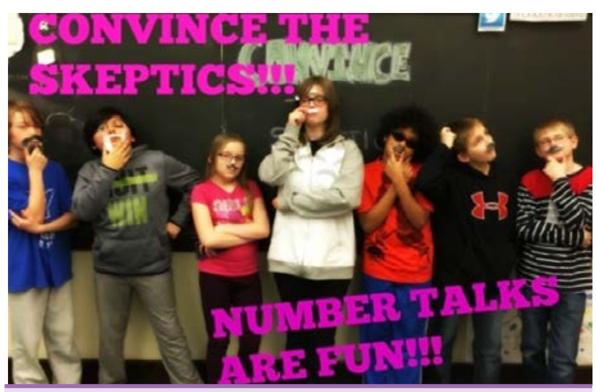


Photo 6. The Accountable Number Talks Success Criteria and Learning Goals (left). Math Journal prompts (right).

2. Creating a Non-Threatening Classroom Environment

Creating a safe culture in which students would feel open to share and address problems with one another was something Suzana identified as a need for the group. She took some time to help students better understand the role of a skeptic in order to help foster citizenship skills in the learning environment. This included dressing up as a skeptic, posing for a photo shoot, creating dramatic role play scenarios, and reflecting on these experiences during a **KB Circle.**



Students dress up as "skeptics" as part of the learning taking place around community-building and citizenship skills in Number Talks.

Suzana provided a deep integration of Character Education in the learning community throughout all subject areas studied. The group of students became the leaders of the Positive School-Wide Culture Initiative, as they created the hashtag #choosekindWHB with student voice concurrently deepening their individual knowledge of specific traits, such as Respect, Optimism, Compassion, and Kindness.

Visual conversation bulletin board showing individual student contributions.

As the culture grew stronger, the number of student contributions to the group discussions increased. As accountable Number Talks/KB Circles progressed, Suzana documented and summarized each valuable student contribution and posted them to a visual conversation bulletin board about 3 to 5 times a week.

3. Democratizing Knowledge: The T.O.G.A. Table!

With anticipation of developing a collaborative learning space while transforming the environment, Suzana placed a whiteboard on top of a double table positioned at the centre of the classroom. Students gathered around this central table to share, reflect, explore, and create ideas during their Number Talks/KB Circles. Suzana identified the success and significance of this new space, which served as the Number Talk/KB Circle anchor area. Suzana encouraged the class to develop a name for it. The students coined the name T.O.G.A. (Table of Great Achievement). This marked a memorable moment during math class, as the students felt a sense of connectedness as they giggled, and chanted its name alongside their teacher.

Suzana added new KB Scaffolds gradually to the T.O.G.A. table as they arose in the students' discourse. At the initial stages, the discourse of the Knowledge Building Community is supported by only two scaffolds on the T.O.G.A. Table:

"I agree/disagree with ____ 's solution because…"

Over time, however, several other scaffolds emerged and naturally became a part of the community members' dialogue. These **scaffolds were written down on coloured paper and pasted along the outside of T.O.G.A.** to create accessible visuals for students. Students became increasingly empowered to contribute to the shared goals of the **Knowledge Building Community** (see Photo 7).

Photo 7. The T.O.G.A. table

THE TEACHER GRADUALLY RELEASES RESPONSIBILITY IN THE DISCOURSE, PROMOTING PEER-TO-PEER MATHEMATICAL INTERACTIONS.

The students were encouraged to reflect on the success of their KB Circles regularly. They clearly identified that everyone's idea was needed and desired, and **Idea Diversity** occurred naturally during their KB discourse. As students gained the confidence and ability to take pride in contributing to collaborations during a study of fractions, decimals, and percentages, Suzana turned over the class discussion entirely to the students. It was a huge Aha! moment for Suzana, to finally release her responsibility during Knowledge Building Circles. The students had arrived at a destination where they could freely explore big ideas and value one another's strengths throughout the learning process.

4. Constructive Use of Authoritative Sources

Students were encouraged to use and evaluate source materials to further refine their ideas around decimal theories and principles. With the help of Dr. Monica Resendes, Suzana provided an expert vocabulary word cloud that highlighted key mathematical terms extracted from authoritative sources, such as textbooks and Ontario Curriculum Guides. The students were also provided the opportunity to view a word cloud based on their own vocabulary that was generated on Knowledge Forum (see Figure 9; for more on Knowledge Forum see pgs. *96-108*). This was a non-evaluative assessment for students to connect mathematical terms to their ideas, and to deepen their understanding of the mathematical procedures with the vocabulary during investigations and communications. Students used this feedback to further explore and refine their understanding of mathematical concepts that additionally resulted in the use of more mathematical vocabulary when justifying solutions in mathematics.

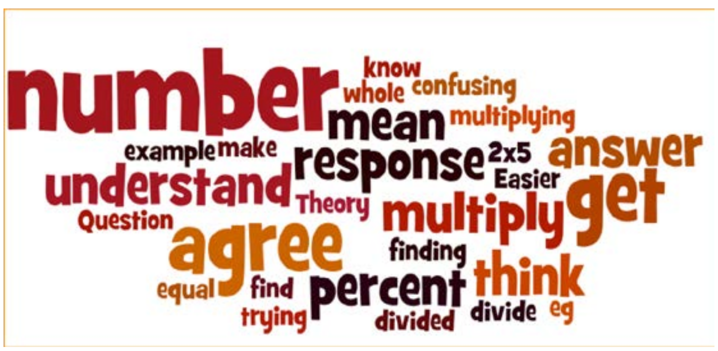
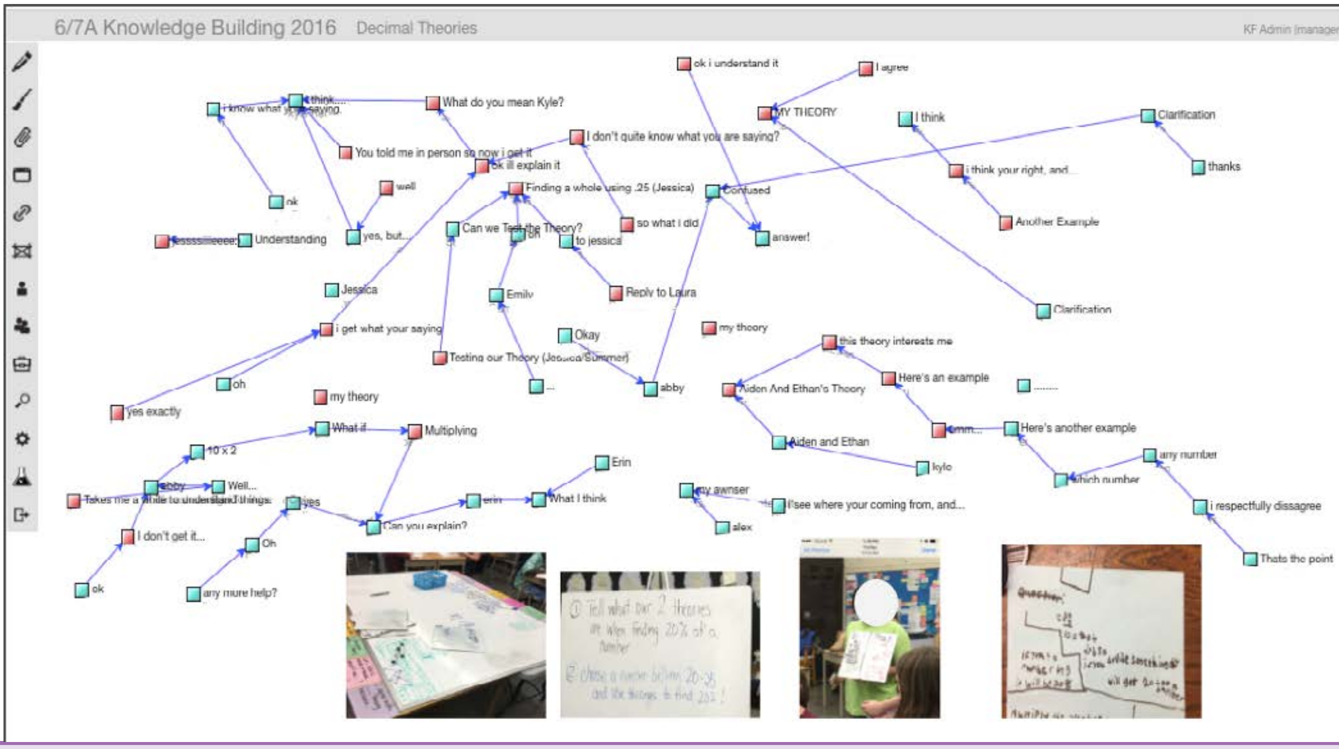


Figure 9. The Knowledge Forum generated word cloud showing student vocabulary (top); the expert word cloud depicting vocabulary from authoritative sources (bottom).

> **What was your "Aha!" moment?** Releasing the responsibility during Knowledge Building discourse was a great moment. Allowing for students to find their own place in the Knowledge Building community was something I had to learn to be patient with by allowing for it to naturally occur. I had modelled, encouraged, and lead the students to this point of discourse, and didn't quite let go until another educator, Denis Maika, pushed my thinking by suggesting this idea. The students reveled at the idea of engaging in the KB circles without my presence. As I stepped out, students were able to further explore theories around the mathematics. The transition was seamless, and this proved that the Knowledge Building community was advancing knowledge effectively.



Students discussing their "Decimal Theories" on Knowledge Forum. The red and blue squares represent notes that open up when clicked. Red notes have been read, while blue notes are yet to be opened and reviewed. Images from classroom work are lined up along the bottom to spur ideas and thinking.

5. Assessment and Evaluation

Using data collected from assessments such as Student Journal Reflections, and Accountable Number Talk/KB Circle observations, Suzana provided feedback to students on an ongoing basis. She met with guided groups to explicitly teach concepts students were struggling with, as identified in journal entries, observations, and conversations that took place both in Accountable Number Talk/KB Circles and on Knowledge Forum. The assessment tools Suzana used, including an Accountable Number Talks/KB Circle Rubric and an Accountable Number Talk/KB Circle Student Reflections Rubric, can be found on the following page, as well as in the Assessment Exemplars (see pg. *141*).

6. Next steps

Suzana's next step will be to focus on increasing **Democratization of Knowledge** by fostering greater student-to-student connectedness in the KB community. Student achievement will increase in mathematics as she continues to provide opportunities for her students to connect math pervasively. It is expected that KB in mathematics will expand so that it not only involves the single subject with the set particular group of learners. Involving experts, parents, and even students in the greater global community will deepen all stakeholders' knowledge, understanding, and values in mathematics.

ASSESSMENT TOOLS:

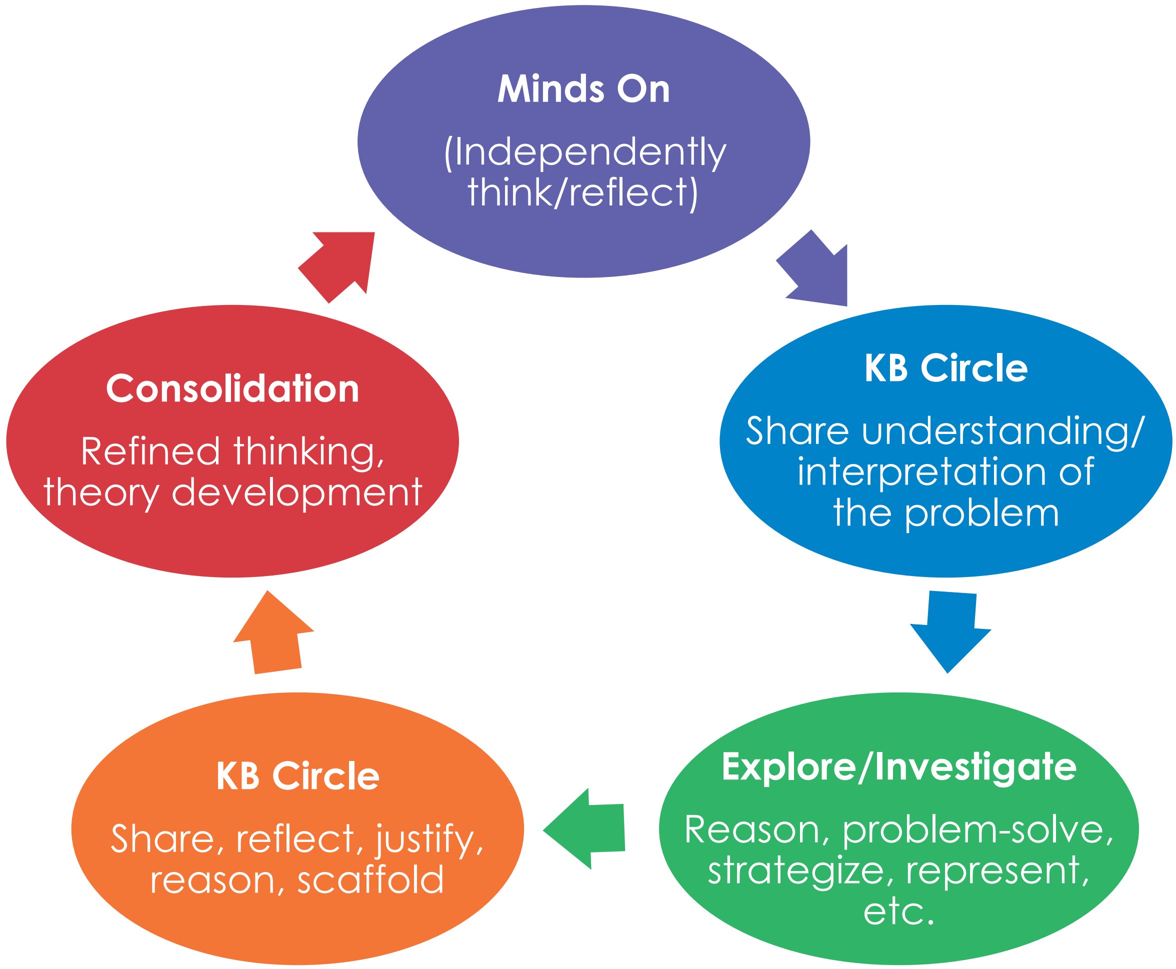
KB Circles/Accountable Number Talks Rubric				
Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Student demonstrates knowledge of math content during Number Talks	Student demonstrates little understanding of number concepts when sharing information during Number Talks	Student demonstrates some understanding of number concepts when sharing information during Number Talks	Student demonstrates an understanding of number concepts when sharing information during Number Talks	Student confidently demonstrates an understanding of number concepts when sharing information during Number Talks
Student expresses mathematical thinking with clarity and logical organization when communicating in number talks	Student expresses thinking with clarity and organization with limited effectiveness	Student expresses thinking with clarity and organization with some effectiveness	Student expresses thinking with clarity and organization with some effectiveness	Student expresses thinking with clarity and organization with a high degree of effectiveness
Student communicates orally to justify a mathematical solution, or express a mathematical argument, using mathematical vocabulary	Student communicates using mathematical vocabulary with limited effectiveness	Student communicates using mathematical vocabulary with some effectiveness	Student communicates using mathematical vocabulary with considerable effectiveness	Student communicates using mathematical vocabulary with a high degree of effectiveness

KB Circles/Accountable Number Talks – Student Reflections Rubric				
Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Student expresses mathematical thinking with clarity and logical organization when communicating about number talks in written form	Student expresses thinking with clarity and organization with limited effectiveness	Student expresses thinking with clarity and organization with some effectiveness	Student expresses thinking with clarity and organization with considerable effectiveness	Student expresses thinking with clarity and organization with a high degree of effectiveness
Student communicates in written form to justify a mathematical solution, or express a mathematical argument, using mathematical vocabulary	Student communicates using mathematical vocabulary with limited effectiveness	Student communicates using mathematical vocabulary with some effectiveness	Student communicates using mathematical vocabulary with considerable effectiveness	Student communicates using mathematical vocabulary with a high degree of effectiveness

RECAP: NUMBER TALKS

Number Talks where teachers intentionally use KB Principles lead to a culture where students:

- hypothesize mathematical ideas
- test and tinker with mathematical concepts
- make mathematically convincing justifications
- listen and respond to mathematical ideas/strategies/theories
- build on others' ideas/strategies/theories
- clarify their understanding
- reflect and apply new learning, ideas/strategies/theories



The case study on the following page illustrates how mathematics was emphasized and integrated within a Knowledge Building study. Note the emphasis on collaborative **KB Discourse,** and **Community Knowledge, Collective Responsibility** throughout the process. (Watch the video series Math in Action here to see this class in action; or, check out the KB Case Studies package at http://thelearningexchange.ca/wp-content/uploads/2017/04/Knowledge-Building-All-Case-Studies-Accessible.pdf for this and more math-related case studies).

CASE STUDY (GRADE 7/8): KNOWLEDGE BUILDING IN THE INTERMEDIATE MATHEMATICS CLASSROOM.

Paula Molloy is an Intermediate Mathematics teacher at John A. Leslie Public School in Scarborough. The following case study describes the experiences and activities that her students participated in while they considered themselves Agents of Change. The Intermediate students explored relevant issues such as: Crime Rates in Toronto, Muslim Discrimination, Genetically Modified Organisms, Teens and Screens, Gender Inequality, Poverty in Toronto, The Consequences of Drinking Bottled Water on the Environment, and more **through the strand of Data Management and the framework of Knowledge Building.**

INTRODUCTION

Having a background in Early Childhood Education, I have spent many years observing children and how they acquire knowledge. Culturally, we seem to understand that children learn from their environment and the stimuli that we provide for them. We freely and naturally provide young learners with tools with which to build, create, and problem-solve. We prepare an educational environment that supports open-ended approaches to learning, encourages a variety of problem-solving strategies, and assesses and honours the process of learning.

I have adopted these same practices in my Intermediate mathematics classroom. Not only has this process involved investigations such as building structures with embedded algebraic expressions, or creating picture books that tell a story about the birth of a mathematical concept and the mathematicians behind the concept, but we extend also our sense of wonder into inquiry projects that make connections beyond the classroom and into the global community.

STARTING WITH THE KB PRINCIPLE(S):

- Real Ideas, Authentic Problems
- Epistemic Agency
- Community Knowledge, Collective Responsibility

KB Provocation: Real Ideas, Authentic Problems

As students mature they continue to make sense of their world. The Intermediate student is a curious learner who is able to receive and synthesize information, and form and share their own perspectives. As a class, we discussed current events and global issues that impacted their lives. The provocation started with a discussion around the election process in the United States and Donald Trump's perspectives on Muslims and immigration. Conversations were filled with emotion and personal narratives, as connections were made and opinions shared.

Students were asked to select an issue that not only provoked emotion, but one that they were genuinely interested in learning more about, with the belief that they could better educate themselves to make more informed decisions or even create change for the future. They needed to think about how information was presented to them mathematically. Learning goals and success criteria were collectively constructed and the curriculum expectations were clearly identified.

Real ideas and authentic problems allowed students to acquire relevant knowledge, making sincere and meaningful connections. This not only informs the learning but may inspire and motivate students and give students the skills to challenge and change the world around them.

Epistemic Agency

Students collectively or independently determined the issue that was of importance to them; similarly, they had autonomy in determining their work teams. In establishing a culture of Knowledge Building, it is imperative that teachers lay the foundation of what collaborative communication looks like and sounds like. For example, it was reinforced that we are all learners and that all efforts in moving forward are to be built and expanded on as a collective. Students were reminded that all team members had an equal role to play in establishing the inquiry question and participating in the Knowledge Building process. Further to this, team communications and responsibilities were to be documented in a Communications Book in every class. When conferencing with groups, I would refer to the Communications Book and reference individual contributions and their documented process of how the team was building and developing suggested ideas, as well as them having a greater understanding of their next steps.

Strategies to Sustain Idea Improvement: Community Knowledge, Collective Responsibility

As students worked throughout the inquiry process, they needed to work through the challenge of what it looked like to research data independently and to share and learn from it collectively. For example, students were often referencing data that were presented on charts and graphs. Although graphs may have been collected by different team members, the analysis of information and the conclusions made were collectively accomplished. Team members may have developed their own survey questions but, collectively, they discussed the purpose and quality of those questions before establishing a final survey. Collectively, they made inferences on the results of primary and secondary data. Collectively, they made predictions, based on their data, about the future of their issue. Collectively, they discussed how their generation could play a role in shaping the future of their issue. Students were heard discussing the need for governments to change standards and policies on issues such as increasing penalties for criminals, greater consumer education on genetically modified organisms, the consequences of drinking water from plastic water bottles on the environment, and so on.

Students participated in a practice that required the contributions of all team members with the expectation that all ideas would be validated and collectively refined.

Improvable Ideas

Within the culture of the learning environment, the process of learning is where the growth, development and building takes place. All ideas are valid and must receive recognition. If the team decides that an idea is productive to their overall goal, then the idea must be nurtured in order for it to develop and expand. Upon observation during the inquiry process, I could hear a team with a focus on gender inequality attempting to work through a problem with a comparison that they were making. One of the team members was quietly offering a suggestion that could move the thinking forward, allowing the group to narrow their comparison of data. The team, however, remained focused on their current train of thought and were unresponsive to the student's improvement plan. As an observer and a facilitator, I asked the team to explain their challenge and then I asked them to once again listen to the improvement plan of their peer. It was a learning opportunity that served to not only push their idea forward but also to reinforce that all team members must be heard when sharing ideas and improving upon them.

In terms of the mathematics, for example, groups often experienced difficulty with primary data collection. They were faced with the challenge of attempting to collect accurate and representative data within their own school. The growth was observable as teams worked through and improved their ideas, ensuring that they were asking bias-free questions that allowed the person being surveyed to answer honestly, and sometimes even anonymously, to ensure the most accurate results. Similar discourse took place when faced with the challenge of selecting appropriate graphs to represent particular data. Students improved their ideas through discussions that included the purpose of communication for each graph type.

The use of the Communication Book was also a valuable tool as it documented the process, development and improvement of their initial ideas, and allowed students to reflect on the rich journey that lead them to success.

Embedded, Concurrent and Transformative Assessment

During our mathematics inquiry, the assessment was embedded within the process. The process is the learning. I had regular and ongoing communications with individuals and with teams. I anecdotally recorded observations that also included the support and direction that I gave to each team. Students continued to document their process within their Communications Book, paired with occasional progress reports.

Providing feedback throughout the process was critical. It was my role to ensure that students were maintaining a clear focus, asking and researching rich questions, working equitably as a team and referring to the learning goals and success criteria that were collectively established. Throughout this process I was able to push the students' thinking forward. Students were asked to go beyond researching information on their issue and ensure that they were bringing their own informed perspectives to the project. For example, the team that was comparing gender inequality in Canada compared to Pakistan speculated that it was likely that information regarding issues of inequality and abuse perhaps is not reported and that the actual data may indeed be more concerning than is mathematically documented.

The growth and achievements made throughout the process need to be assessed. The process is where the thinking, problem-solving and refinement take place. The product is a result of the process; therefore, the assessment, embedded within the process, and the finished product and presentation, all need to be assessed.

Conclusion

The conclusion of our inquiry included a reflection piece. Students must reflect on their journey and their process of learning. A reflection should highlight their challenges and discuss how challenges were overcome, their surprises, their successes, the knowledge that they built as well as the new curiosities that arose from their inquiry.

With this particular inquiry, a part of the concluding process was actually the birth of a new and thought-provoking journey as students identified the possible future. Based on the actual data within their inquiry projects, students were to make informed predictions about what their issue would look like 10 or 20 years from now, with the realization that they are indeed the Agents of Change. We abstractly placed these supported predictions into a time capsule and acknowledged that the behaviours of their generation would shape not only their own future, but also the future of the next generation. As an educator, reflection is equally as important as we continuously strive to refine our own ideas, take risks, maintain an open mind and provide best practice methods that will further the quality of learning for our students.