

WHAT WORKS?

Research into Practice

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Research Monograph # 34

How do students learn
how word problems work?

Word Problems

Connecting language, mathematics and life

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Research Tells Us

- Students often struggle with word problems because they treat them too realistically and suspend their own sense-making abilities.
- Students from lower-socio-economic backgrounds or minority cultural or linguistic backgrounds tend to have greater difficulties with word problems.
- When writing word problems of their own, students are able to mathematize real-world scenarios.
- The challenge is to draw on this ability, so that students can respond appropriately to word problems they have not seen before.

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To be successful in solving word problems, students need to learn how to read such problems. Simply decoding words or extracting arithmetic operations is not enough: students must learn to read between the lines and understand what they are expected to do mathematically. Consider the sample problem on the Grade 3 EQAO test:

Steven earns \$5 for every bundle of newspapers he delivers. He wants to buy a game that costs \$18. How many bundles of newspapers does Steven need to deliver to earn enough money to buy this game?

Who is Steven? How many newspapers are there in a bundle? What is the game he wants to buy and why is it so cheap? For an EQAO test, such questions are not appropriate, yet students certainly ask them as they struggle with the challenge of relating the real-world context of the problem with the mathematical task.^{1,2,3}

Often, students combine the numbers in the problem in apparently non-sensical ways or give unrealistic solutions. For example, students might find that Steven must deliver 3.6 bundles, rather than a more realistic four bundles. Such responses have been attributed to a kind of “suspension of sense-making.”⁴ Sometimes the issue for students is that word problems are not realistic, but “stylized representations of hypothetical experiences,”⁵ leading them to treat word problems *too realistically*, introducing considerations that are not appropriate such as specific experiences of newspapers or children named Steven.

How, then, can students be supported to make sense of word problems so that they can treat them successfully as mathematical problems?

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Solving word problems realistically (but not too realistically)

Students from lower socio-economic backgrounds or from minority cultural or linguistic backgrounds tend to be less successful in solving word problems, often because they treat the problems too realistically.^{6,7} Such students can struggle to draw appropriately on realistic considerations in their thinking.

Solving a word problem can be seen as a simple process of translating words into a mathematical expression and then solving the problem. Verschaffel, Greer and de Corte⁴ argue that this “closed” approach, in which students are only expected to use the information given and a specific mathematical method, is simplistic and leads to the suspension of sense-making. They argue for an “open” approach in which students draw on several sources of information to tackle problems in a more realistic way. This approach can be characterized by four main steps: 1) understand the problem situation, 2) mathematize the situation, 3) conduct a mathematical analysis and 4) interpret and communicate results.

In the classroom, students working on the EQAO sample problem cited earlier might: 1) research rates of pay for paper routes and costs for different games; 2) mathematize the situation in terms of repeated addition, repeated subtraction, division or multiplication; 3) complete their calculation using a variety of methods; and interpret their results in several ways and communicate them to other students.

This approach has much in common with the broader notion of problem solving referred to in the Ontario mathematics curriculum. Students who use this open approach to word problems will be more adaptable and more successful in solving previously unseen problems, such as those encountered in test situations.⁴

What genre (linguistic form) are word problems?

The genre or linguistic form of word problems is rather peculiar. Most word problems exhibit the following general features:⁸

1. They have a three-part structure: scenario, information and question. In our EQAO example, the scenario involves Steven, his newspaper route and a game. The information consists of \$5 for a bundle and \$18 for a game; the question asks how many bundles Steven must deliver.
2. The information is arbitrary in relation to the scenario. Steven could be paid \$8 for each bundle, and he could be saving for a skateboard. In terms of how students interpret the words to identify a mathematical task, the problem would be the same.
3. They involve ambiguous use of verb tense, time and reference. It is curious, for example, that Steven *earns* rather than *earned* \$5 for each bundle. The question is set in the timeless present, implying that the he is still delivering newspapers. Several aspects of the scenario are merely implied – we are not told explicitly that he delivers newspapers, only that he earns \$5 for each bundle. These features may be particularly challenging for English language learners.

What do students pay attention to?

Examining how students write their own problems gives some insight into what features they are aware of, not just as a mathematical task, but also as a form of text. In one study, I recorded pairs of Grade 5 students working to write multiple problems. My analysis of these recordings, including the following extracts, shows that students pay attention to at least three dimensions of word problems: genre, mathematical structure and personal experience.^{9,10}

Students pay attention to genre ...

Courtney: I have a hundred and fifty cars in my business. Group these into three groups. See if you're clever.

Zeb: That don't even make damn sense.

Courtney: Yes it does.

Zeb: What's the question then?

Courtney: Huh, group hundred and fifty into three parts.

Closed Versus Open Approaches

A closed approach is simplistic ...
Students are expected to translate words into a mathematical expression, often leading to a suspension of sense-making.

An open approach is realistic ...
Students are encouraged to draw on several sources of information to tackle the problem in ways that make sense to them.

Courtney's initial idea suggests he is aware that word problems need a scenario. The students almost always started by thinking of a scenario, rather than a mathematical equation. Zeb's response shows his awareness that word problems typically include a question. Courtney's suggestion, "see if you're clever," suggests an awareness of one purpose of word problems.

Students pay attention to personal experience ...

Safia: I know [...] one thousand people were at the mall, no, at Ikea.

Rahim: (overlapping) yeah, Ikea.

Safia: Ikea is big, last time I went there it was too crowded.

Safia draws on her experience of the furniture store to justify the choice of numbers in her problem – she is making the scenario meaningful. This kind of reference to personal experience occurred frequently during the study, sometimes as brief comments, sometimes as extended discussions of topics like pocket money or going to the supermarket. These discussions were always related to the students' word problems.

Students pay attention to mathematical structure ...

Courtney: Okay then, there're a hundred an' eighty brains in the morgue. [25 second pause]

Jackson: Mm. Ninety gets took out how much left?

Courtney: No no no, no no that's take away then. Half, what's half o' ninety, forty-five? Four, four monsters came and took, an' ate, ate, forty-five each.

Jackson: Huh?

Courtney: Four monsters came and ate forty-five each – so, yeah if four monsters came and ate forty-four each.

Courtney and Jackson discuss the mathematical structure of their problem quite carefully, including what operation and numbers to use and how the numbers relate to each other: They are engaged in valuable and meaningful mathematical thinking. Again, this kind of discussion arose frequently, often leading to modification of the problem, suggesting that students are able to relate the words of a problem to a mathematical operation.

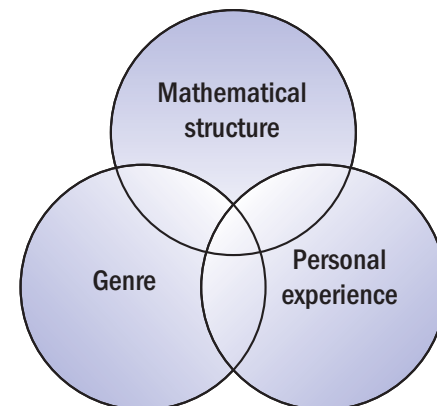
These three aspects of students' attention are interrelated. Personal experience is used to make the underlying mathematical structure meaningful and to interpret the scenario of the word problem. Understanding the structure of word problems is necessary to successfully mathematize the scenario. Students are quite able to mathematize situations based on real-world considerations when writing word problems of their own – the challenge is to draw on this ability, so that students can respond appropriately to word problems they have not seen before.

What are the implications for classroom practice?

Relating genre and mathematical structure

- Ask students to compare two problems with different scenarios but the same mathematical structure. Discuss the differences and similarities between the two, noting that the scenario can be changed while leaving the underlying mathematical structure and calculations unchanged. For example, compare these two scenarios:
Steven earns \$5 for every car he washes. He wants to buy a skateboard that costs \$18. How many cars does Steven need to wash to earn enough money to buy this skateboard?
Steven earns \$5 for every bundle of newspapers he delivers. He wants to buy a game that costs \$18. How many bundles of newspapers does Steven need to deliver to earn enough money to buy this game?
- Ask students to compose word problems for a given mathematical calculation (e.g., $30 \div 5 = 6$). Compare the different problems and their solutions, and discuss the different ways in which the underlying mathematical structure is represented in words (e.g. grammar, syntax, vocabulary).

Students pay attention to three dimensions of word problems ...



Implications for Classroom Practice

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- Give students opportunities to practise mathematizing word problems; ask them to rewrite the problem or or change the mathematical operation required. For example, given an addition problem about a barbecue, ask students to transform the problem into a multiplication problem while maintaining the barbecue scenario as closely as possible.

Each of these activities should be followed by some discussion of the language used in students' word problems and how that language relates to the underlying mathematical task.

Relating personal experience and mathematical structure

- Invite students to revise a given word problem to make it more comprehensible or more realistic. For example, given a word problem set in a shopping scenario, students could research current prices and update the problem accordingly.
- Ask students to compose and solve word problems relating to recent events in their lives, such as a trip, shopping expedition, or hockey game. It is important that these events be elicited from students.

Each of these activities should be followed by some discussion of how the students' word problems are constructed and which aspects of the students' experience and/or perhaps other parts of the curriculum are relevant when solving these problems.

Relating personal experience and genre

- Invite students to engage with problem contexts by extending them in different ways (e.g. given a problem about a garden design, ask students to create a picture of the garden; given a problem about a school trip, ask students to prepare a short drama scene or write a story about what happened on the trip).
- Have students relate their work to the solution for their original problem.

Following each activity, discuss the role of a scenario in a word problem, including the idea that some details are not relevant to the mathematical task.

In Sum

Word problems can be challenging for students. An open approach to word problems should engage students in understanding, mathematizing, analyzing and communicating in the context of meaningful situations and problems. Authentic activities like those suggested in this monograph promote a greater understanding of how word problems work as texts. In the same way that an understanding of the structure of stories makes students better readers of stories, so an understanding of how word problems work as a genre can make students better readers and problem solvers.

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