WHAT IS SPATIAL REASONING?

>> Spatial reasoning is really I think two things. One is the kind of reasoning that we do when we try to navigate around the world and find our way. So that's complicated and difficult and it's its own kind of challenge. You don't want to get lost and lots of us get lost a lot. I don't think that's the kind of spatial reasoning that people have in mind when they think about the link between space and math. There's another kind of spatial reasoning that is kind of smaller scale, more focussed on objects, more thinking about what does this object look like if you turn it -- that's often called mental rotation. Thinking about how long things are, how much they weigh, how much area there is, what they would look like if they were folded or bent or twisted or rotated. So those kinds of spatial reasoning, I think, are the ones that are at play when we think about math. Of course it looks different at different ages and with different kinds of mathematical subjects that you want to get across. So in preschool there's often a focus on shapes, and I think that's important, but often people don't really think about how to teach shapes. So if you look through the sort of shape book that a three year old might have, or a shape sorter, they're always the little canonical perky triangle, equilateral usually with a pointy side up. And that really doesn't give you the idea of a scalene triangle or a weird skinny, tilted triangle. It doesn't give you anything about the formal definition of there's three lines and they come together. So that's an example where even I think at a very simple level a spatial concept that is also a numerical and geometric concept should be taught, but we need to think more about how to teach it. It's also important to adjust how you teach it to the level of the kids. So a preschooler wants to play. They don't want to sit still for you to lecture about, "Well, there's three sides and it doesn't matter what angle they come together at," and this kind of thing. But you just arrange the book or the shape sorter to embody these different kinds of principles. At a much higher level, you might be talking about a missing addend problem. Now it's not algebra, although it actually in a way prefigures algebra. But when a fourth grader is dealing with a missing addend problem, what they often do is take the equal sign as an invitation. "Okay. Add up everything on the left side. Stick it in the answer slot." And they ignore that, well, there's this other number that they're supposed to also take into account. So that's where I think you can see that there's a spatial underpinning in that there's a concept of balance that is quite spatial, and of the numbers as having a kind of weight that has to balance. There's also a symbolic aspect because you need to remember or know or have been told or discover that the equal sign isn't about, "Okay. Give me an answer now." It's about equal. And you can see that I just can't resist making a gesture to really communicate that, and I think to me that shows the, you know, spatiality of the concept.

SPATIAL THINKING

>> I think for each kind of mathematics and each kind of space there might be different relations. One, and I already gave a few examples in terms of shapes and missing added problems. Another example is the number line, where I think the continuity between the integers is very important. And it gets neglected early on in our haste to teach the integers and simple addition and subtraction. I think it would be good to support all along the intuition that children do have

when they're in preschool, but they somewhat lose as they learn to add and subtract. The intuition of continuity. So if they didn't forget about continuous quantity, I think they would have an easier time with fractions. And I think they would also have an easier time with negative numbers. Because again, the number line is just the extension, sort of what's on the other side of zero. When you think of what a negative number is, people often refer to money. Like well I owe you money, well that's negative numbers. But another way to think about it is the other end, the other side of zero. And then you can also go to this balance concept that I talked about before. So that also allows you to put flesh on the bones of things like if you subtract a negative number you're really adding. I mean when you say it, it just sounds like gibberish right. Like a minus minus is a plus. Like really. Well why would that be? But if you think about it in terms of a number line, you can sort of think of it as well you know, you're being dragged over here, well or over here, depending on like, I should speak to you the negative numbers are here. But then if you're taking that away it's like you're allowed to bounce back over here. So the whole thing makes sense in a more embodied action oriented way. So I think for each of these mathematical concepts we can think of the spatialization of it. I think that encouraging children to make sketches as they solve word problems you know, we often do that. But not always. And sometimes we don't have enough space on the paper for them to make sketches, and that can be helpful. There are some cases where it's not helpful. Once you make a sketch and then you look at it and it's wrong and you know, it's there. You can exit out but it's all a mess and you know. So I think you have to experiment with the use of sketching. Sketching is helpful, sometimes more so when it's linked with talking to yourself about why you're sketching. So it's not that spatial excludes the verbal, the spatial can be merged with the verbal. So you're explaining, maybe just to yourself, maybe to a partner, what it is that you're putting down on paper and why. Another kind of example in teaching involves the spatial dimension of when you're working with examples. So say you have two ways of solving algebra equations. Now we're talking a little bit older kids. If they're lined up horizontally so you can point to the correspondences between them, then that builds generalization about what it is that you're doing as you solve a problem and what choices you're making. If you have two problems and they're lined up vertically, it's actually much harder to draw the correspondences because in going from this line to that line you're sort of having them skim over so many other lines that it's visually confusing. That, it sounds complicated when I explain it, but really there are some very basic spatial elements, they are almost graphic design elements, to think about when you do that kind of a comparison.

PEDAGOGY

>> The idea of direct instruction is often regarded as very old fashioned. I think that the idea of inquiry based or playful learning is sometimes regarded as more modern, although it's been around for a while. But they're definitely regarded as polar opposites. And I think I would prefer to say that they are both important, that neither works well without the other. The direct instruction gets very boring. Kids learn things by rote. There's drill to kill, and they get really turned off. Those are the downsides. But the upsides are that there are some things that are very hard to discover on your own. After all, it took Newton [inaudible] to discover calculus. If you

want people to just discover it on their own that could really take quite a while, and many people would never discover it. So the flip side is that we really do have to do some direct instruction to the extent that we can link it with some inquiry base, with some enjoyable problems, with communicating what the context is of why you would want to know this math anyway and why it's useful, as they stretch their you know, newfound abilities and look for new domains of application that can be inquiry based. They can work in groups. All of those things are good things. But I really think they're meant to work together. Fractions are related to continuity and they're related to the number line. So I think actually that the concept of quantity that we come with when we're babies is a concept of continuous quantity. I think the concept of integers is sort of extracted out of that and really doesn't start to emerge until three, four, and then it gets very reinforced in the curriculum, which stresses the operations of addition and subtraction focussed on the integers. So fractions are back to continuous quantity. And teaching fractions involves kind of resurrecting what you always knew, which is the idea of a quantity between the integers.

SPATIAL LANGUAGE

>> The obvious is true. Kids who play more with blocks and play more with puzzles develop better spatial skills. One of the things that parents do naturally when they interact with their children as they play with blocks and puzzles is talk to them. So it's not just you buy them, you put them in the playroom. Interaction is also important because then you're using spatial language, and the spatial language directs their attention to aspects of the blocks and puzzles that are going to be important. And you might say, "Well, I don't know what spatial language is and how would I know if I was doing it right?" But you're just going to do it naturally. If you talk to a child about a puzzle, you're going to say, "Well, this is flat. And the edge of the puzzle, we need the flat bits for the edge of the puzzle." Those are spatial concepts. Those are spatial words. Now you might not use them. You might say, "Well, the tree is at the top. So can you find some green bits?" And concentrate on the colour. But you're going to use some spatial words. Use those play materials. Use them naturally. Move up the difficulty level so that your child stays engaged, but not so it's so hard that the child is dismayed and just watches you do it. There's just some very commonsensical principles. And that extends up into elementary school if you can engage the child in building more complex models, those kinds of things. Gestures are very helpful in all sorts of kinds of learning. Since they are spatial in that they happen in space, they're very natural in the mathematics curriculum. I think the first thing to remember is that it's instinctual to gesture, and you shouldn't keep yourself from gesturing if you're a teacher. And you shouldn't worry about your kids gesturing. You don't want to have this kind of well ordered classroom where everyone just keeps their hands folded on their desk. So just encouraging children to gesture is important, and it turns out when you look at kids, some gesture more than others. And doing any kind of gesture is probably helpful as opposed to not. So you don't have to get too technical here. Just use your hands. And your hands you always have with you so you don't have to think about, you know, fancy manipulables or anything like that. There are some things that researchers have been probing, though, that I think give us some food for thought. So a gesture is an action, but a gesture is also an abstraction away from an action. So you're not actually lifting and pulling and touching and interacting. You're almost miming or referring to that

kind of an action. So a gesture is a good tool for thought because of that abstract nature of the gesture. So on a continuum from just doing where we might not even know why we're doing what we're doing, we're just doing it, and very abstract symbols and words and so forth, a gesture is really a bridge between those two.

GENDER

>> Mental rotation is one of those skills, that mental twisting of an object in your mind that shows pretty big sex differences and yet paper folding. If you take a sheet of paper and fold it up and maybe punch holes through it and unfold it and where are the holes? Really doesn't, maybe very small ones. Well, there both about objects and mental transformation or why would one show sex differences and the other one not? So, that's where I think we, well A, we have a very deep lack of knowledge. B, what I want to say is people skip too fast to well, why, why are boys better at mental rotation, is it environment or is it biology? There again, we do not know the answer. There are some reasons to point to environmental factors, there are possibly some hormonal factors, we do not know the answer. But I think it's important to stress is these are great scientific puzzles. I love to pursue them I think that they should be pursued but the practical point is that we don't need to solve them to do a good job raising our children and teaching our children. Because what we found is that both boys and girls can be engaged, both boys and girls improve. Girls might even become as good as boys to the extent that they improve a lot. So, yeah, let's just do it. Let's just work on it and who cares about the differences basically. There are lots of video games that probably do include spatial skills. Some of them are more navigational skills and we haven't been talking so much about those as you wind your way through the maze and that kind of thing. Some of them are fairly spatial in the sense that we have been talking about so you know, putting things together and so forth. Unfortunately a lot of computer games though, involve a lot of aggression and so that is what draws boys to them. I don't think it's that the girls don't want to do the spatial things it's that the girls don't want to spend all their time shooting bad guys. If we had more women on computer game design teams, I think that would not be the case. The Sims was a computer game, that actually involved women designers and it came out very, very differently from most other computer games. So that would be one of my proposed remedies.

IMPROVING SPATIAL REASONING

>> Training and experience can improve spatial skills a lot. We have recently looked over the literature in this area and sort of done a statistical analysis that puts together the results of all these hundreds and hundreds of studies. And what statisticians call the effect size, the sort of summed effect, overall these studies is at least moderate. And in some cases what you would call large. So it's very, very clear that these skills can be improved a lot. And, furthermore, these -- The improvements last for at least a few months. And honestly they might last longer. People haven't yet had the courage to look at how long they last, but they seem to last at least for a few months. And they also do to at least some extent generalize to other skills. So if you get better at mental rotation, you also get better at paper folding and vice versa. Now how far do they

generalize? Does that also make you better at math? We don't know that. It would be exciting to start to trace it. But I think we have every reason to be optimistic. I think that manipulatives are great. Digital things are probably good too. The one caution I have about digital is that sometimes there's a distracting quality to them so you can spend more time on some of the bells and whistles than you do on the real interaction. Looking at preschoolers again, one of the findings that I've been involved with a team, we're reporting it recently, is that if you're playing with real shapes versus using an app with your three-year-old that involves moving shapes around and fitting them into virtual slots on an iPad, you get more concentration on the shapes and more talk about the shapes with the real shapes than you do with the digital shapes. And I think that's because they're really there. So I -- I'm cautious about the digital, even though I know that it can be great. But you have to think through how you do it.