All students are capable of doing mathematics, and all students have the right to engage in rich mathematical tasks. All students have the right to develop their own solutions to problems, and all students actually need to be valued as mathematics thinkers. But sometimes, I think, you know in actual practice, we sometimes forget about that. Sometimes we think, well, they can't really engage in problem solving until they have the skills, but actually, it has been shown that students who one might call struggling learners benefit the most from having problems where they are allowed to show multiple strategies, and they are allowed to think a little bit differently about it. We know that if we have a really rich task, that it actually has multiple entry points, and those multiple entry points allow everybody to engage in it. It might be building a model, trying a diagram, but somehow everybody has a starting point in that problem, and I think it is really important that all students have the right to engage in meaningful mathematical activities, not just memorizing skills and procedures. I think we have to be somewhat careful about learning goals and success criteria. We know that there is evidence that suggested it helps to improve student achievement. When they know sort of why am I doing this? But I think we have to be fairly careful that we don't make them too narrow. On the one hand, I am talking a lot about opening up space for students to develop different strategies, and different ways of doing that. If we open up that space, and at the same time, we have a learning goal that keeps them here, this is the method that we are aiming for, we've got a conflict going on. So I think in the work of designing learning goals and success criteria, we have to make sure that we make space within those. You know, when I think of some of the work we do in criterion-referenced work, or learning goals, we think about targets. Not my favourite word. And we think about all kids have to get to a particular place, and I don't think that is true. I think all kids need to move their thinking along, and need to expand their own mathematical thinking. And people will get to different places on different timelines. So I believe the success of a particular lesson is that at the end of the lesson, if I look at every individual student in that class, and if I think about where their thinking was before, we engaged in this lesson, and where their thinking is now, if it has moved along, I think it is a successful lesson. So I think we need to also pay attention to individual student thinking. Maybe at the end of the lesson, so what kinds of things have we learned? Because, well, our goal might be that students will work on this problem using proportional reasoning. I have seen them work on that problem using something else, and it is still a rich lesson. And we still have moved that mathematical thinking along. So I just think we have to be careful, to make sure that we set up goals and success criteria that makes space for students' thinking.

Actually hard to talk about assessment sometimes without actually talking about teaching and learning, because I think that previously we thought about assessment as an event. It's something that occurs at the end of a unit and the end of a year. But with the ways that we're thinking about assessment in terms of formative assessment and summative assessment where people might say, assessment as of and for learning, we
know that sound assessment is actually embedded in instruction. It is ongoing. It's constant. So in other words, every single move that a teacher makes if they're really paying attention to student thinking, if they're noticing the ways that kids are thinking about math, that's assessment, and it's happening all the time. So, the way a teacher questions, the way they listen to the student thinking and the way that they respond to that thinking is actually ongoing assessment, as well as other particular kinds of things that the teacher deliberately sets up; so for instance a quiz that they might provide feedback on, as opposed to merely giving a mark on. That's a form of assessment that helps the teacher understand how the students are thinking, and it also helps to inform the student in terms of their thinking and what kinds of moves they should do next. Sound assessment is critical to me in terms of it improving student achievement, and it's not just to me. The research is pretty clear about that, that formative assessment has a very high impact on student achievement. So the fact that it--- We also involve students in the assessment process. So it's not something that's done to them. It's done with them, so when I mean working with kids around assessment, they're co-constructing Rubrics, they're co-constructing success criteria, they're engaged in peer and self-assessment, and they're taking some of the feedback from the teachers, and learning how to be able to actually adapt that to change the way that they're thinking about things. So I think assessment is critical, but it's also as I said, a difficult thing to talk about separately, because it's really connected to the teaching and learning. Assessments are designed for different purposes, and, you know, so often, particularly large scale assessments that are designed for the purpose of accountability are often used by others to do a variety of things that they should not be used for, because once we use them for purposes that they're not designed, it really questions the validity. As I say that, I also think that while we talk about formative and summative assessment, there are actually times when assessments that are done---considered summative are sometimes used formatively. And that's actually not a bad thing. So in other words, if we think about a large scale assessment like EQAO, and if that's done at the end of the year, teachers actually use that data to think about how can they then inform their own teaching practice as they move forward. So in a sense then, summative assessment is used formatively. So, it's sort of a bit of a blurring of the lines sometimes, and we have to do it fairly carefully, but I think we have to recognize that those sorts of things happen.

PLANNING MOVES FOR TEACHERS

>> It's a tricky task teaching to tell you the truth. So, I think that when we had, I'll just call it traditional teaching, I don't know it could be the kinds of lessons you and I sat through when we were students. Where it felt as though it was what I called the 'old school three part lesson', you take up the homework, you give some examples and then you do some more homework. That isn't a easy lesson to prescribe. It's easy to talk to new teachers about what that looks like. But we know that's not the most effective way for kids to learn mathematics. It might work for some but it doesn't work for all. So instead, what we're doing is asking teachers to possibly select a rich task, anticipate the responses, monitor the student thinking, select in sequence the kinds of strategies that we think would be most effective for kids to share. And I don't mean the most effective
strategies, I mean choosing those kinds of solutions that allow us to have a rich discussion and make connections between the strategies. That's tricky-- but with experience and with working collaboratively I think that helps. I also find that there are cues around pedagogical moves by looking at the curriculum. So in other words, and a lot of work that I do with teachers and administrators around the curriculum is, let's look at the curriculum and let's look at the verbs in the curriculum. Because the verbs actually tell us what action should be occurring in that classroom; because the verbs are not just add, subtract or solve. The verbs include things like demonstrate, explain, compare, determine through investigation all of those kinds of verbs actually help to tell me what those pedagogical moves are. So they help to tell me what the structure of that lesson might actually look like. And that, what sort of setup I need, what sort of tasks I need, what sort of questions I might be asking. The other thing around looking at the verbs and the curriculum that I often point out particularly to beginning teachers, is that when it says demonstrate, compare, explain; it says the students will demonstrate, the students will compare, the students will solve, the students will explain. It's not about the teacher doing all of that. And so, in a way that sounds easy but it's actually difficult because you need to set up a context where students are doing that and where you are actually listening to that. Particularly when I'm working with new teachers, one of the things that I talk to them about is, when they design a lesson, I often look at it and I say, "so within the 75 minutes or 40 minutes whose actually doing the math" because quite often teachers feel as they should be the ones doing the math but it's actually the students who should be the ones who are engaged in math during that amount of time. And that might sound really easy because you think well then, what am I to do as a teacher but it's actually quite difficult, because it's about setting up that environment where the kids do engage in the meaningful mathematics in that activity. And where the teacher's able to ask the right questions, consolidate the learning, connect the different mathematical ideas. So the task is actually quite a bit more difficult. Students arrive in class and they don't no idea about how much planning went into some of these lessons. One of the books that I often refer people to is the book called the '5 Practices' by Peg Smith and Mary K. Stein. And I refer to them to that because it talks about five practices to facilitate a meaningful mathematical discussion. As much as I think we can't prescribe what that class looks like, I think teachers need some guidelines in terms of what that looks like. And so what they discuss as practices are things such as anticipating student responses, so doing the problem yourself, thinking about the different ways students would solve it; and then in-class monitoring the ways that students are actually working through the problem, taking notes, paying attention to it; all the time thinking about how can I connect these strategies or help students connect it. And then it's about selecting and sequencing those particular solutions that should be shared not because they're the best necessarily but because they will bring out those important mathematical ideas that the teacher wants to be able to discuss in that class. And then finally to connect, so to take those ideas that have been brought out and put on the table; and to help to connect those sorts of ideas. So I think that, that sort of thing helps to give people some guidance as to, so what does this really look like? I know in some of the work I've done with administrators and quite often principals want to know, what do I look for in a math class? Because you know it's not about just kids being engaged, I mean they could be engaged if we brought in videos and pizza, right? That's what they; we do at a birthday party. It's about being
engaged in meaningful mathematics and what I encourage for principals to do is, same thing I tend to say, let's look at the curriculum. So the "look for's" are actually there if the teacher is addressing an expectation that says students will demonstrate their understanding of the equal sign through and I ought to see kids demonstrating. So I think that it's important, it's not just about having manipulatives on the table and having kids in groups. It really is about, so, what is the math that's going on here? And what are the mathematical actions that should be going on here? Sometimes people ask me questions about, well how much should be skills and how much should be problem solving? But it's so much more than that. It's not that dichotomy of one over the other. And it's all of those mathematical actions that, that you know, I've been talking about, that actually should be occurring.

LEARNING THE CONTENT KNOWLEDGE

>> In many cases teachers were taught in a very procedural way, taught mathematics in a very procedural way, which means that their understanding of those particular concepts might not be as robust as they would like them to be, to be able to facilitate these ideas and make those mathematical connections. There's a lot of evidence that talks about teachers being able to deepen their own math understanding through their own teaching as long as they are open to trying some of these ideas. If one kind of sticks to a textbook and just worries about right answers, then you're not opening up that discussion. Sometimes they can't do that alone either. So it's about networking with other teachers. And a lot of the work that I've seen in terms of professional learning, that really helps to support the teacher's own, what we call in the research world math knowledge for teaching. So understanding math in ways that help teachers choose appropriate representations. Here, the mathematics within student thinking, et cetera. A lot of that work has been done within professional learning communities where teachers might get together and let's say do a lesson study. So they might say okay. Let's look at the concept of integers and how that's developed from Grade 7 to 9. And let's look at the curriculum expectations and let's co-plan lessons that we might teach in Grade 7, 8 and 9 and then let's go see how it plays out within Grade 7 and how the lesson plays out in Grade 9 and how it plays out in Grade 8 and then they can start to understand a variety of things. So one another's context is one. I find that quite often secondary teachers are quite surprised at the culture of Grade 7, 8 classroom and 7, 8 teachers are quite surprised about the culture of Grade 9 classroom or school. So there's that. They get to know one another's world. But they also look at how those concepts are developed over time. They go deeper into those ideas rather than just a quick lesson. And I find that that's one way of helping to develop not just their pedagogy but their understanding of the mathematics. But there are many other ways to do it. Examining student work is a really good way for teachers to engage in developing their own understanding of mathematics. Because I often, for instance, show quick little videos of students thinking, and the conversation that comes out of that is very surprising. Quite often the teacher is surprised at the way the student solved it because it wasn't the way they would solve it. And as students they start to question that then they start to expand their own understanding of math. I find that the more we listen to different ways that people solve problems the more it helps us develop our own understanding. So I have
hope that that can be done. And I have -- you know, I see evidence of that. We all see evidence of that, as teachers work with their students and with their colleagues to plan really meaningful lessons. I think we need to recognize that many of the system leaders are in a similar situation to many of our teachers in that they learn mathematics in a very procedural way. And I think sometimes when they need to see themselves as instructional leaders in mathematics, that can be a bit challenging for them as well because many might be more comfortable in terms of supporting literacy moves rather than mathematics moves, again, through no fault of their own. Some of the kinds of things that we’re talking about in terms of what teachers need to do are also the kinds of things that I believe principals need to do; so to work collaboratively, to deepen their own understanding of math knowledge for teaching and to really think about so what does good teaching and learning in mathematics look like. And I know across the province there are a lot of principals who are engaged in doing just that and are working in that. You know, when we think about professional learning, see, I see all of this as a bit of a fractal. So I see it as kind of nested communities of inquiry. We see the classroom where the kids are engaged in inquiry. We see teachers and they’re engaged in inquiry, and they kind of keep bringing back that new knowledge through the classroom and the classroom knowledge back to their own professional learning community. And then I also see systemwide people or school administrators also involved in professional learning, moving in and out of, sort of I make it look like a hierarchy but it's a bit different moving in and out of sort of their school communities, working with their teachers. Good learning is good learning whether we’re talking about students or teachers or kids, and working together to work on not just mathematical ideas but pedagogical ideas. I think it's really best done in a meaningful community of inquiry. One, where people are working on the kinds of problems that make sense to them and that they actually want to work on. I know sometimes we talk about professional learning communities. And well, they seem to work well, so let's set one up and here's what we're going to do with it. And that's not what I'm talking about. Teachers can decide what ideas they want to work on to enhance their own learning and then it tends to have more meaning to them.

PARENTS AS PARTNERS

>> There are many different ways. And I know sometimes parents -- it's hard on parents. And I think we need to do a better job of communicating with parents about what really is going on inside mathematics classrooms. And a lot of that work I think is starting to occur through things like family math nights where a school invites the parents and the kids to come together and do mathematical activities. And I think that really helps parents to understand. So what's happening in math class, you know, and it gives teachers an opportunity to talk to the parents about no, we value the different ways. The kids actually engage in this problem, and it gives parents a chance to see why we might be using concrete materials in math class and how they actually help us solve a problem. Because sometimes they're not getting those messages from their students. They're getting different sorts of messages such as, you know, 'so what did you do in math today?' 'Well, we played with blocks', you know. And parents might not understand that actually they were engaged in rich tasks where the blocks were used as
mathematical thinking tools to help them solve the problem. So one thing I think schools need to do is to set up better communication with the parents and invite them in to take part in those sorts of things. But also then I think it gives parents a lens into oh, so when we're actually baking at home, we're doing math. Or when we're having them help us build a fence and doing some measuring, they're actually engaged in math. And I think that's an important thing for parents to recognize that they're engaged in math. Math has kind of a bit of a bad rap in society and I think we need to work on that. One of my previous PhD students her PhD was actually looking at what are elementary students views of mathematics? And what kinds of messages are they getting from the media and from society in general? And one of the things she had them do was to draw a mathematician. So you can imagine what that looked like. Einstein here, pocket protectors, short pants that show their white socks and basically E equals MC squared out of a chalkboard. So I'm not sure too many 12-year-olds really want to become that. So if that's the image of mathematicians and mathematics. I think we need to think about changing that a little bit, and we need to make sure that everybody is, recognizes that they are capable mathematicians.

GETTING KIDS MOTIVATED

>> One of things I really value is the different ways that people and not just kids finding their ways about solving math problems. But the way teachers take up engaging their kids in mathematics. Because it's done in a variety of ways and all of those ways make sense. I think one of the biggest challenges is actually allowing teachers, or letting teachers recognize what kinds of mathematics kids come to the table with. I think that quite often kids are turned off mathematics, because it's as though they come into class and the math that they actually do outside in the world is sort of dismissed. And I think that quite often they see math that is something that is merely rules and procedures. And not really connected with their world. So I think one of the challenges is for teachers to be able to think about, and recognize, and listen to the kinds of mathematical ideas kids come to the table with. We're all mathematicians, we're born mathematicians, we think mathematically, we count numbers, we are able to use some intuitive sense in order to be able to solve problems and I think that needs to be recognized. So, not only that prior knowledge but connecting to the kinds of experiences that kids have. Math anxiety often comes about because people expect or see mathematics as having one right answer and one right way to actually solve the problem. And that's not true at all. Well sometimes there may be a particular right answer and we can't disagree about that, we do want people who are flying planes to recognize that their accuracies kind of important. There are many different ways to actually approach and to come to that particular answer or that particular solution. So I find sort of traditionally math is seen as something that you need to do quickly and do in particular ways. But it's not about speed. It really is about understanding, so being able to give kids space. To develop their own strategies for solving problems, to be able to think about different ways that they can combine numbers or different ways that they might be able to come up with an equation at the higher levels in terms of, sort of the ways they think about a problem, are really important it takes the pressure off, it gives them space to be able to solve some problems and to bring their ideas to the table. So I
really think taking out that issue of speed and that issue of people sort of having to do math in a particular way gets rid of that anxiety. I know in one research project that I worked on, where teachers were encouraged through professional learning, to allow students to develop their own strategies and to work with those strategies. One of the biggest things that the teachers found at the end of this two year project is that they noticed the biggest difference with the students that they would have characterized as struggling learners. That those learners actually felt as though they had a place at the table. That the ways that they came up with solutions were valued all of a sudden as opposed as being told they were wrong and not the traditional method. So it's those kinds of things and those ways of teachers listening to the ways kids think about problems that I think are important. In terms of differences in terms of the way kids think let's say well I think one of the biggest differences, I think that we're actually letting them think. So I think that's sort of an important thing. And I think that a lot of the work I do is research with teachers about their practice and quite often when their talking about shifting their practice to make space for students to solve problems in different ways, they're actually astounded at the ways that kids come up with to solve problems. And some of them are solving the problems using concrete materials, they're drawing diagrams, they're creating charts and tables and I think it is important not to privilege one way of solving a problem over another, I think sometimes people feel that if they're using symbols it's a better solution than a diagram. I've actually seen some really rich tasks solved beautifully with a diagram that much, have a much better representation of the problem than an algebraic solution that seems a bit too, too removed from the actual problem. So I think when we talk about their patterns of thinking, once we open it up anything is possible and I actually think that's one of the challenges for teachers. So in other words if one teachers in a way where there's one right answer, one way of doing it, then it feels a little more confined for the teacher to be able to work with. Once we open it up and allow anything, then I think it can a bit more challenging for teachers, they need to be able to think about trying out the problem themselves, anticipating the different ways the kids could interpret that problem and the different ways that they could possibly solve it. And that could be a bit of a task for teachers and to be open to the fact that they may not anticipate all the ways that a student could solve the problem. They may walk into class and be struck by some other solutions that are very unique, and I think sometimes teachers worry well what if they come up with something and I'm not sure if it's right. And I think we have to let that go. And we have to allow the kid to explain their thinking and allow other kids to listen to that thinking and make sense of that thinking. So it's risky, it's a situation that requires a willingness to take some risks to do that. But I think it needs to be celebrated.