REPORT

A Forum for Action

Effective Practices in Mathematics Education

December 11 &12, 2013

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Ontario Ministry of Education

Table of Contents

The thoughts expressed in this report are not necessarily those of the Ontario Ministry of Education, but instead reflect those of the participants in the Deember 11 and 12 forum.

Executive Summary

Mathematics – like literacy – is considered a "fundamental," not just for some children but for all. So it is of concern that there has been a small but gradual decline in assessment results for mathematics in English language boards at Grades 3 and 6 over the past five years. Additionally, while gains have been made in Applied Mathematics, the gap between results for students in Grade 9 Academic and Grade 9 Applied remains significant.

Against this backdrop, the Ministry of Education designed a two-day conversation – a Forum for Action – to mine the best thinking of researchers and practitioners alike with the goal of improving mathematics teaching and learning in Ontario schools. The key considerations emerging from their discussions are presented below (in no order of importance):

1. We need to continue to work together – the Ministry of Education, school districts, faculties of education and teachers –in acquiring deeper content knowledge of mathematics and improved pedagogical knowledge and skill.
2. Increasing teachers' deep content knowledge in conjunction with their pedagogical knowledge will require knowledgeable and skillful facilitators working with them -- hands-on – in school and classroom settings.
3. School districts require multiple approaches to sharing promising practices – both within and across schools – for intervening with students who are struggling in mathematics.
4. A team approach – where teachers plan, learn and teach together – is vital for improving mathematics teaching and learning on a wide scale.
5. Teaching mathematics effectively requires a range of strategies, including the purposeful design of lessons, a careful sequencing of student activities and appropriate interventions and scaffolds for student learning. It also requires knowing students well to work with the strengths, needs and interests they bring to school.
6. Improving mathematics outcomes requires building the confidence of both teachers and students in their ability to go deeper into learning mathematics.
7. Spatial reasoning is an important predictor among early learners of later success in mathematics – visual representations are essential for developing spatial reasoning and supporting growth in student understanding of mathematical concepts.
8. Manipulatives that help illustrate mathematical functions and computations are excellent and appropriate tools for all student learning K–12 and for teacher and administrator learning as well. When manipulatives are used regularly, they help learners move from a concrete understanding to a more abstract one and back again.
9. Many students with learning difficulties benefit especially from sustained use of manipulatives along with precise instruction that meets their individual learning needs.

10. An area for continued research and development is the use of technology in mathematics. Technology can be helpful for many students who have specific learning difficulties and its use for all students needs to be explored.
11. Quality early learning in mathematics is essential as a foundation. We need to become systematic about quality early years experiences in mathematics.
12. Staffing is an important variable and strategic staffing – placing the highest skilled teachers where needs are greatest – has proven to be valuable in improvement efforts.
13. Mathematics programming needs to involve time for the development of procedural fluency complemented by building strong conceptual understandings.

Taking Stock to Improve Outcomes for Ontario Students

Researchers describe mathematics skills as a “new currency of modern societies around the world” (OECD, 2010, p. 33). In our highly complex and evolving technological world, we rely on mathematical data, projections, inferences and systematic thinking, so much so that mathematics – like literacy – is considered a “fundamental,” not just for some children but for all. Science, technology and many of the arts are based in mathematical elements and not surprisingly early achievement in mathematics is a leading indicator of later academic success across subject areas (Duncan, 2007).

So, it is of concern that while Ontario is recognized as a high-achieving school system by international standards (Brochu, Gluszynski, & Cartwright, 2011; Mourshed, Chijioke, & Barber, 2010), on provincial assessments, English language boards overall have experienced a small but gradual decline in mathematics scores at Grade 3 and 6 over the past five years. Additionally, while gains have been made in Applied Mathematics, the gap between results for students in Grade 9 Academic and Grade 9 Applied remains significant.

This is the backdrop for the Ministry of Education’s recent Forum for Action, a two-day conversation designed to surface ways to improve mathematics learning in Ontario schools.

The forum brought together leading mathematics education researchers with educators representing a small sample of boards and schools that are showing improvement in some key areas of mathematics. Their task was to get the ideas flowing – for consideration, discussion, challenge, reflection and further pursuit – to the end of improving outcomes for all Ontario students.

To help forum participants take stock, Dr. Bruce Rodrigues, Chief Executive Officer of Ontario’s provincial assessment office – Education Quality and Accountability Office (EQAO) – shared key results from the most recent provincial assessments, as follows:

EQAO Results at Grade 3 / 6 / 9 in Brief

Keeping in mind:

- EQAO assessments are based on the Ontario curriculum – the tests are designed to allow students to demonstrate what they can do vis-à-vis the curriculum. It’s not a contest!
- EQAO assessment results are only one measure that schools have to use. It is recognized that classroom teachers are in a position to have the deepest understanding of what their students understand and what they can accomplish.

Key results:

- 67% of Grade 3 students met or exceeded the provincial standard[1] in 2013, down from 70% in 2009.
- 57% of Grade 6 students met or exceeded the provincial standard in 2013, compared to 63% in 2009.

[1] The provincial standard is a higher standard; it is set at “B” grade level.

- In 2013, nearly 1 in 5 students did not meet the provincial standard in Grade 6 despite having met it in Grade 3.
- 84% of Grade 9 students enrolled in Academic math met or exceeded the provincial standard in 2013, up from 77% in 2009.
    - Among students enrolled in Academic math, 93% that met or exceeded the standard in both Grades 3 and 6 met it again in Grade 9.
- 44% of Grade 9 students enrolled in Applied math met or exceeded the provincial standard in 2013, up from 38% in 2009.
    - Among students enrolled in Applied math,78% that met or exceeded the standard in both Grades 3 and 6 met it again in Grade 9
    - However, it is worth noting this represents only the 19% of students enrolled in Applied math that had met or exceeded the standard in both Grades 3 and 6.

Some generalizations:

- Students who meet the provincial standard early in their schooling are more likely to maintain higher achievement in secondary school.
- Students who do not meet the standard early are more likely to continue not meeting it in later grades.
- While the most recent results for the EQAO Applied math assessment represent an improvement, mathematics continues to be a very challenging subject for many students enrolled in the Applied math course.

Adapted from the forum presentation of Dr. Bruce Rodrigues
http://mathforum1314.files.wordpress.com/2013/12/bruce-ministry-math-conference-2013-2.pdf

Mining the Research in Mathematics Education

Assistant Deputy Minister, Mary Jean Gallagher, set the context for the research discussions, introducing 11 distinguished academics mostly from Ontario universities with deep ties to district school boards. Each researcher was asked to address the following question prior to the forum in a brief research report and to explore that same question with the 100 or so participating educators who have experienced some recent success in their mathematics results:

> "What has your research revealed about the effective learning or teaching of mathematics?"

The researchers' reports and presentations and video vignettes from the forum are available at http://learnteachlead.ca/en/projects/a-forum-for-action-effective-practices-in-mathematics-education-december-11-12-2013

A number of key themes from their presentations and discussions with participating educators emerged, as follows:

1. Teaching mathematics involves developing the ability of students to demonstrate complex thinking.

As discussed by Dr. Ruth Beatty, Lakehead University

Dr. Ruth Beatty teaches the mathematics methods course for pre-service Primary/Junior teacher candidates in the Faculty of Education at Lakehead University (Orillia). Her seven-year research study of how children learn complex mathematical concepts, particularly in the domain of "early algebra," has been supported by a number of federal grants and has resulted in a comprehensive course of study for students in Grades 4 to 10.

- Effective instruction includes working with visual representations of mathematical processes, sequencing lessons and providing students with opportunities to make conjectures and justifications.
- Students who work primarily with visual representations (including concrete patterns, diagrams and graphs) are more successful at understanding algebraic relationships, finding generalizations and offering justifications than students who are only taught to manipulate symbols or memorize algorithms.
- Providing sequenced, interrelated activities gives students opportunities to construct mathematical understanding by bringing together their theories, experiences and previous knowledge. Sequenced tasks, coupled with open-ended activities, allows all students (including struggling learners) to access aspects of mathematical thinking that are normally challenging
- The pace of introducing new concepts should be driven by the rate of students' developing understanding. Effective instruction includes a large assortment of interrelated and sequenced activities, the use of visuals as well as numeric and symbolic representations and multiple opportunities for practising procedures and reviewing concepts and skills.
- The sophistication and complexity of student mathematical thinking increases with practice and skillful instruction. Also important are the actions of offering conjectures and providing justifications.
- Making conjectures is a vital skill to extend student thinking. For example, rather than focusing on the results of specific calculations, for example, 2 + 2 = 4, students can start to make conjectures about the properties of numbers. They can explore the outcomes of adding different combinations of odd and even numbers: even + even = even, odd + odd = even, odd + even = odd and make conjectures about the patterns they are noticing.
- Providing justifications is just as important a skill because it encourages students to provide reasoning and evidence to validate their thinking as they engage in mathematical problem solving. With practice, students learn to move beyond justifying their solutions based on specific examples to providing justifications based on more deductive reasoning.

2. High-quality mathematics programming in the primary years builds the foundation for future success.

As discussed by Dr. Joan Moss, University of Toronto

Dr. Joan Moss is an Associate Professor at the Jackman Institute of Child Study in the Department of Applied Psychology and Human Development at OISE/University of Toronto. Her research has focussed on the design and assessment of curricula for children's development and understanding of rational numbers and early algebra. More recently she has worked as one of the Principal Investigators on the Math for Young Children Project (M4YC), with a focus on the teaching and learning of geometry and spatial reasoning in Kindergarten-Grade 2 classrooms.

- Early mathematics is a key predictor of student academic success, not just in mathematics but also across disciplines (Duncan et al., 2007).
- More and more evidence is showing that geometry and spatial reasoning are foundational to later success, not only in mathematics but also in related fields such as science and the arts.
- Spatial reasoning at age 3 predicts mathematics performance at age 5 (Verdine et al., 2013).
- By the time students enter Kindergarten, there is already a wide social economic status disparity in their math readiness. This gap appears not only in developing numerical understandings, but also in student spatial reasoning abilities. Gaps appear early and widen throughout the early school years.
- Of importance, however, is the evidence that shows that spatial reasoning is trainable at all ages (Uttal et al., 2012) and that early interventions have the greatest and most lasting effects (Heckman, 2006).
- Evidence from the Mathematics for Young Children Project (M4YC) suggests that young students can engage with complex mathematical concepts with sustained attention and are far more capable than previously thought.
- The M4YC approach to professional development is an adaptation of Japanese Lesson Study (Lewis, Perry & Murata, 2006). Results show that M4YC teachers broadened their own understanding of concepts central to geometry and spatial reasoning and increased understanding of the breadth and depth for the specific content area in focus.
- A "playful pedagogy," envisioned as a kind of guided play for young children as described by researchers (Fisher et al., 2010), informs the M4YC approach.
- The features of playful pedagogy include engaging and playful contexts which involve narratives, fantasy scenarios, and/or games, challenging and important areas of mathematics content, authentic exploration and inquiry, open-ended questioning, engaging circle formations and students sharing and learning from their peers.
- This kind of pedagogy involves the best of play-based learning which encourages curiosity, choice and autonomy, but moves beyond to include supporting engagement with broad curricular goals and encouraging children to think beyond their own self-initiated exploration.

3. **The ability to represent and compare numerical magnitudes is an important foundation for the learning of mathematics.**

As discussed by Dr. Daniel Ansari, Western University

Dr. Daniel Ansari is an Associate Professor and Canada Research Chair (Tier II) in Developmental Cognitive Neuroscience in the Department of Psychology and the Brain and Mind Institute at Western University (London, Ontario). Ansari's research focuses on gaining a better understanding of how children develop numerical and mathematical competencies, why some children fail to acquire basic calculation skills (developmental dyscalculia) as well as what brain circuits are associated with the processing of number and our ability to calculate.

- Children need a fluent understanding of the meaning of number symbols before they can use them to perform mental operations with symbols, such as calculation.
- The basic ability to represent and compare numerical magnitudes appears to be an important competency to have early on in the learning of mathematics.
- Connection/mapping between symbolic and non-symbolic representation of numerical magnitude is particularly important.
- Activities that help children understand the ordinal relationships between numbers may help children understand symbol-quantity relationships as well as symbol-symbol relationships.
- Researchers believe that the time between Senior Kindergarten and Grade 2 is a sensitive period for the development of fluent symbolic number processing.
- "Developmental dyscalculia" is a specific learning disorder that is being researched by Dr. Ansari and colleagues. Children with this disorder experience significant and persistent difficulties in learning basic numerical and mathematical skills, while they may be performing well in other domains such as literacy.

4. **The purposeful use of technology can build both spatial reasoning and number sense, important skills in early math development.**

As discussed by Dr. Nathalie Sinclair, Simon Fraser University

Dr. Nathalie Sinclair, Canada Research Chair in Tangible Mathematics Learning, is tapping into young children's natural abilities in spatial reasoning to find ways to engage and motivate them to learn through physical activities and visual perspective exercises. Sinclair is also examining how computer-based learning can increase children's mathematical sophistication at different ages. Emerging technologies are making it possible for children in younger grades to interact with potent mathematical concepts.

- Research findings indicate that children come to school with a great deal of informal geometric understanding and there should be more emphasis on spatial reasoning in the curriculum to capitalize on student capability.
- Dr. Sinclair's research has found that the teacher's promotion of mathematical reasoning and well-designed tasks can help students to develop important concepts such as symmetry,

tessellations, two-dimensional shapes, angles, parallel/perpendicular lines and so on. She suggests that such programs such as *Sketchpad* are highly effective for children in this development.

- Integration of more advanced geometry at an earlier age would provide continuity with higher levels of the curriculum.
- Well-designed digital technologies can help children learn and express themselves in ways that do not depend solely on written communication and therefore may invite a broader range of ways of thinking.
- Major challenges in the use of technology in the mathematics classroom relate to the need for teachers' professional learning. Primary teachers are often not aware of effective resources and so do not make "purposeful use of technology" and do not necessarily know what the important mathematical ideas associated with a given task might be.
- Very little research exists on how to support teachers' effective use of appropriate digital technologies and this is an important area for development.
- As there is strong potential for technology use in mathematics education, professional learning for teachers should include support in identifying software suitable for problem solving, discussion and exploration. Offering teachers a small number of recommended tools so they can be used purposely is a very useful start.

SEEING LIKE A MATHEMATICIAN

5. There is a diversity of ways to support the active learning of mathematics.

As discussed by Dr. Walter Whiteley, York University

Dr. Walter Whiteley is a researcher in Applied Geometry (in many disciplines), Geometry Education and Spatial Reasoning. He is the Coordinator of Graduate Diplomas in Mathematics Education at York University and of the MA in Mathematics for Teachers program. He also developed and directs an Honours undergraduate Mathematics for Education major for pre-service teachers of mathematics. He served as the York University representative and co-chair for the Council of Ontario Universities (COU) Mathematics Curriculum Review Task Force during the last round of revisions of the secondary mathematics curriculum.

- As learners, we combine approaches and multiple representations into an increasingly powerful network that becomes "the way we think" in mathematics
- We have access to some of the ways that we develop mathematics skills from the use of our senses and we continue to use our senses for reasoning and representations. The appropriate use of kinaesthetic and visual/spatial senses should be a part of classroom activities. When they are, we build connections, helping us to re-connect with concepts.
- Materials, including concrete manipulatives and appropriate technology which support different entry points to learning, are essential to supporting the diversity of students in our schools and

the diversity of ways that mathematics can be learned and applied. Manipulatives are appropriate for all levels.

- Spatial reasoning is a key ability and a predictor of success in Science, Technology, Engineering, Arts and Mathematics – what is referred to as the STEAM disciplines. In our current curriculum, many students leave high school with clear deficits in spatial and kinaesthetic reasoning which has been identified as a barrier for retention and success in Engineering programs.
- There is no single trajectory that is best for all students. This results in a need to differentiate instruction.
- Classrooms with rich, multi-disciplinary problem-solving activities can engage students deeply.
- Many pre-service teachers, and many in the general public, appear to believe that procedural learning must come first to then support the richer relational or conceptual learning, which comes later. This is not what classroom research shows. Results show a focus on procedural learning is limiting, often leading to shallower knowledge and disinterest in deeper learning.
- Mathematics classes should be ones where students are able to ask questions that matter to them regarding how mathematics connects to their world.

DESIGNING EFFECTIVE INTERVENTIONS FOR GRADE 9 APPLIED MATHEMATICS

6. Collaborative teacher inquiry makes a positive difference for both students and teachers.

As discussed by Dr. Douglas McDougall, University of Toronto

Dr. Doug McDougall is Professor and Chair of the Department of Curriculum, Teaching and Learning at OISE/University of Toronto. He has been an elementary and secondary school teacher of mathematics and computer science. He has taught in both pre-service teacher education and graduate programs. His scholarship, teaching and creative professional activity focus on mathematics teaching and learning, collaborative teacher inquiry, the professional learning of teachers, technology-supported learning and academic leadership.

- McDougall and colleagues have developed a conceptual framework to guide improvement in mathematics instruction – The “Ten Dimensions of Mathematics Education Framework” – which breaks down the essential components of a successful mathematics education program.
- Components of the framework include program scope and planning, meeting individual student needs, constructing knowledge (e.g., using questioning to build student understanding), using manipulatives and technology and providing transparent and authentic assessment.
- The Ten Dimensions Framework was used recently to inform a highly successful Collaborative Teacher Inquiry project in Grade 9 Applied Mathematics in 12 GTA schools.
- The selected schools were very low-achieving in Grade 9 Applied EQAO results – their “collaborative inquiries” focused on ways to use assessment, technology, teaching strategies, student tasks, data analysis and so on to improve student learning.
- Of note: as teachers changed the teaching-learning dynamic in the classroom, students experienced more success and demonstrated more self-confidence.

- A collaborative inquiry approach assisted in changing teacher beliefs and practices in the schools.
- Student-centred approaches which focused on building engagement and understanding made the most positive difference for both students and teachers.
- Successful interventions were implemented in the context of a combination of some or all of the following elements:
    - Job-embedded learning
    - Collaborative (peer) inquiry
    - Attention to student performance
    - Institutional and administrative support
    - Provision of time and other resources
    - Commitment to continuous long-term engagement in professional development initiatives

BUILDING TEACHER UNDERSTANDING OF EFFECTIVE MATHEMATICS INSTRUCTION

7. **Knowledge of mathematics and knowledge of mathematics for teaching are both key to teacher impact.**

As discussed by Dr. Alex Lawson, Lakehead University

Dr. Alex Lawson is an Associate Professor of Mathematics at Lakehead University. Her expertise is in how we learn mathematics. She is interested in the way math is taught in our education system, particularly in the primary grades. She has completed a seven-year longitudinal project following the mathematical development of 50 children from the beginning of Grade 1 through to the end of Grade 5 in reform-oriented classrooms. The study comprised detailed video documentation and analysis of their thinking and growth over time.

- Knowledge of mathematics for teaching has two related components: (1) common content knowledge and (2) specialized pedagogical knowledge for teaching
- Lawson and colleagues examined 558 pre-service teachers' mathematical knowledge using a mandatory content exam. The results suggested a gap between what pre-service teachers knew and what they need to know in order to effectively build rich mathematical learning experiences for students. The researchers concluded that many pre-service teachers required further instruction in the fundamental knowledge that would underpin their mathematics knowledge for teaching.
- Teachers' lack of mathematical content knowledge and mathematical pedagogical knowledge is one of the most central issues we face in effective mathematics instruction. Regardless of the instructional methods deemed more or less effective, none can be implemented well unless teachers understand the mathematics underpinning the instruction as well as the related mathematical concepts. Should this knowledge base be an expectation of teacher certification?

- Teachers can facilitate the development of strong student mathematical understanding by beginning with student thinking and supporting student-generated methods to solve mathematical problems. However, in order to do this, teachers also require knowledge of where the student strategies or underpinning mathematical principles lie on a continuum or progression. While the curriculum outlines what should be achieved, it does not outline a progression of evolving strategies, deepening mathematical principles and supportive mathematical models over time. Many teachers will need support in learning to work with this progression as it is likely very different from their own experience of mathematics instruction.

8. Both conceptual understanding and procedural fluency underpin student and teacher success.

As discussed by Dr. Christine Surrtamm, University of Ottawa
Dr. Suurtamm began her career as a secondary school mathematics teacher and department head. She is currently Director of Teacher Education and an Associate Professor of Mathematics Education at the University of Ottawa. She has been the Principal Investigator of a number of research projects that examine the complexity of mathematics and mathematics teaching. She recently served as the Canadian representative on the National Council of Teachers of Mathematics Board (NCTM), Co-chair of the International Congress on Mathematics Education (Topic Study Group on Assessment) and currently is a member of the Ontario Ministry of Education Curriculum Council.

- The Ontario Mathematics curriculum reflects current thinking and research in mathematics education. It is well aligned with the mathematics curricula in other jurisdictions, including those which are high performing in international assessments.
- The Ontario Mathematics curriculum provides a balance between problem-solving approaches and mathematical skill development.
- In reviewing/revising the current curriculum, it would be important to examine the current research on learning progressions. Learning progressions can be described as the possible paths that student learning can take and the types of connections that can be made between different mathematical concepts. These learning paths are supported by empirical evidence.
- The teacher plays a critical role in enhancing student learning. Educators in a curriculum support role vis à vis other teachers, such as math coaches, are important resources and need a strong knowledge base of mathematics and the teaching of mathematics to be most helpful.
- Current thinking and research promote classrooms where students investigate, represent and connect mathematical ideas and develop mathematical procedures within the context of problem posing and problem solving.
- Students develop fluency with mathematics in applied problem-solving that builds upon and reveals the sense of mathematics rules and procedures.
- The emphasis needs to be on developing conceptual understanding along with procedural fluency. Students need to see a set of connected ideas and procedures that they use to reason, argue and engage in mathematical sense-making.

9. Teacher and student efficacy are predictive of and strongly correlated to student achievement.

As discussed by Dr. Cathy Bruce, Trent University
Dr. Bruce is an Associate Professor at the Trent University School of Education and Professional Learning. With 14 years of classroom experience and 11 years in academia, Cathy is passionate in her conviction that all learners are capable of success in mathematics, from the young child to the experienced educator, and that our ultimate goal is to support learners to become capable and creative solution finders for the day-to-day and world-class problems we face. In 2013 she was the recipient of the Ontario Colleges and University Faculty Association Teaching Award, for her excellence in university teaching. Her work is featured at www.tmerc.cal; follow her on Twitter @drcathybruce.

- Teacher efficacy in mathematics is the teacher's belief that he or she can help students learn mathematics, despite professional challenges and despite the individual circumstances and difficulties that students may face.
- Student efficacy in mathematics is the student's belief that he or she has the ability to learn.
- Both teacher and student efficacy can be enhanced through "mastery experiences." These are experiences where one learns something that is challenging and feels a boost of confidence because of this experience – "I can do it this! I am capable of learning math."
- Mastery experiences can be supported through teacher professional learning which has the following characteristics – collaborative in nature, classroom embedded, cyclical in design and asset based – where an ethic of trust, care and accountability prevails.
- Dr. Bruce's research substantiates that a collaborative approach which develops mastery experiences results in improved achievement results.
- Developing spatial reasoning and providing "visual affordances" in both low- and high-technology environments are crucial to help students unlock mathematical ideas, including algebra and fractions.
- Mathematicians work with multiple representations of mathematical concepts, operations and relations and it is very appropriate for students to do so as well – at both elementary and secondary levels.
- A restructuring and reconfiguration of teacher pre-service education programs would be beneficial, providing, for example, mathematics specialization courses for primary/junior teacher candidates, including financial mathematics and statistics, mathematics for early years learners and the integration of mathematics, science and technology.
- Finding ways of using technology to share new knowledge developed by researchers with classroom teachers needs to be encouraged.

10. Teacher concerns and orientation contribute to teacher efficacy.

As discussed by Dr. Jamie Pyper, Queen's University
Dr. Pyper is an Assistant Professor in Mathematics Education in the Faculty of Education at Queen's University and Coordinator of the Mathematics, Science and Technology Education Group (MSTE). Dr. Pyper's goal is to develop and foster a strong and obvious link between theory and practice and to instill

an appreciation of the art and science of teaching where "teaching" is considered to be more about students' learning of mathematics than teacher presentation.

- Dr. Pyper proposes a larger conception of “teacher efficacy” which encompasses: (1) changes in teachers’ concerns, (2) orientations to teaching practice, and (3) perceptions of the internal and external influences on efforts to improve student achievement.
- His research indicates there are cycles of movement through teacher concerns – from concerns about oneself and survival in a classroom to concerns about impact on student learning; concerns about survival in the classroom are linked to lower teacher efficacy and concerns about impact to higher teacher efficacy.
- Teacher orientation relates to the sense of professional knowledge and practice that a teacher has acquired. Dr. Pyper discusses five domains of teacher knowledge, ranging from practical to critical-social knowledge.
- His study of teacher efficacy explores how teachers are negotiating professional learning models such as “collaborative inquiry” and “instructional rounds” (as coined by Richard Elmore and colleagues regarding teachers’ professional learning through classroom observations about a collective problem of practice).
- Attention to the nature of pre-service teacher concerns and orientation to teaching practice may increase teacher efficacy in mathematics and ultimately impact classroom outcomes.

LESSONS FROM FRENCH LANGUAGE SCHOOLS

11. **Mathematics teaching and learning in French language schools affirms the importance of designing a responsive mathematics learning environment.**

As discussed by Dr. Serge Demers, Laurentian University

Dr. Serge Demers is an Assistant Vice-President (Acting) at Laurentian University (Sudbury) and current chair of the Canadian Educational Research Association. Serge has been teaching at the university level for over 15 years, and previous to that taught high school in both northern Alberta and northern Ontario. His teaching and research interests include mathematics education, pedagogical use of technologies, as well as quantitative methodologies and analyses. Serge is the co-author of *Communication et apprentissage : Repères conceptuels et pratiques pour la salle de classe de mathématiques* (2004) and *Processus d’abstraction en mathématiques: Repères pratiques et conceptuels* (2010).

- In considering the relative success of Ontario’s French language schools in the primary and junior grades, studies confirm several key strategies.
- Students have the most success when they work on problems as a group rather than individually. Group work permits students to have meaningful discussions about mathematical concepts. Discussions need to occur between the teacher and the students as well.
- Optimal learning occurs in an environment where the teacher lets students explore concepts but also knows when to intervene and teach the concept directly.

- Meaningful student discussions require open-ended questions or questions that can be solved in different ways.
- Using manipulatives in all grades to solve open-ended questions is helpful for students as they transition between concrete and abstract understandings.
- The use of technology, particularly calculators, permits students to concentrate more on their problem-solving abilities and less on computational abilities; computational abilities do, however, need to be mastered by the student at some point.
- There is a need to explore further the instructional practices that can best promote the transition from concrete and to abstract thinking.
- Further study is ongoing in Ontario's French language schools regarding the sustainability of improvement from elementary to secondary schools.

Learning from District and School Participants

In table discussions, school board teams explored the approaches to teaching and learning mathematics that they felt were making a difference in their schools. Acknowledging the power of research partnerships, they emphasized that joint projects with researchers working alongside educators had led to new learning opportunities for classroom teachers and to promising capacity-building efforts in mathematics teaching and learning at the ministry, board and school level. They reported that students, especially those with persistent and ongoing difficulty in learning mathematical concepts and operations, benefitted as their teachers' knowledge of mathematics became more robust and their instructional strategies more precise.

They also spoke of the value of engaging increasing numbers of teachers in the exploration of effective instructional practice. In this vein, they emphasized the benefits of professional learning experiences that engage teachers in working with their peers to address students' learning needs. Because the vast majority of teaching time is spent with students in the classroom, they underscored the value of opportunities for educators to work and learn together – in professional learning communities, school and district networks, action research partnerships and a range of other inquiry projects. A constant theme was the importance of creating the conditions for rich professional learning – in what educators refer to as a "collaborative learning culture" – aimed at increasing students' learning.

A more in-depth summary of insights from the forum follows.

CREATING CONDITIONS FOR SUCCESS

1. **Belief in the ability of students and staff is at the core of improvement efforts – there is no such thing as "math people" and "non-math people."**

- Educator belief in students' capability and competence in mathematics is fundamental to improvement.
- A student's sense of self-efficacy develops through engaging in successful learning.
- Belief in the capability of staff to support student need and improve instruction is also foundational. As teacher self-efficacy grows, teacher practice also changes.

2. **Student voice and attitudes underlie improvement efforts.**

- Student engagement and student confidence grow as student voice becomes more central.
- Student voice means speaking up, feeling comfortable saying you don't understand something, taking a risk in learning new things.
- In classrooms where student voice matters, where student ideas are integral to dialogue and discussion, students take ownership of their learning.

3. **A team approach is needed to improve teaching and learning in mathematics.**

- Key individuals in boards who have both content and pedagogical knowledge in mathematics are instrumental in building professional learning communities in schools and in networks across schools where learning can be shared. Finding effective ways to "mobilize" this expert knowledge is very important.
- Choosing the area for a team's learning focus may be challenging since time is such a limited resource. Participants talked about trying to find a balance between a variety of topics identified as learning needs in a school and specific topics based on central data analysis.
- A team approach can build greater coherence in programs and practices across grades, divisions and panels.
- When learning opportunities only allow a few teachers to become involved, it becomes a challenge to engage staff in a school-wide effort – especially those who have not been involved in the original opportunity.
- When teachers engage other teachers, this is often more fruitful than a top-down or mandated approach because it helps to build a sense of team efficacy. Additionally, teacher collaboration takes root more easily when the principal and senior staff members are seen as co-learners in a process to develop better ways to teach math.
- Using agreed-upon norms for working together helps to develop trust – trust and sensitivity are needed to engage colleagues who may be insecure about their knowledge of mathematics.

- Training for school and board-based facilitators is also beneficial in the development of a school math team and for broader learning networks focused on mathematics.

IMPROVING INSTRUCTIONAL PRACTICE IN MATHEMATICS

4. School and system leaders play a key role in building capacity for improved instruction.

- Finding ways to build in release time for new professional learning is considered an important leadership responsibility. As one example of why release time is needed for improved mathematics instruction, participants discussed the need to support teachers in designing open-ended tasks that are connected to the Mathematics curriculum, then in analysing student work/thinking on these tasks to identify learning gaps and instructional strategies.
- Strategic staffing was also identified as a strategy worth noting. For example, intentionally placing highly effective teachers in Grade 9 Applied Mathematics classrooms makes a difference in terms of student progress.

5. Clearly identifying learning gaps is the basis for improved instruction.

- The efforts that teachers make in getting to know their students well and learning about their prior mathematics knowledge are important. This knowledge enables the differentiation and personalization that are needed, particularly with students who struggle with mathematics.
  - Some boards are following specific students to track progress – sometimes referred to as "marker students" – with the intention of specifying learning gaps and identifying the early interventions that need to be in place to assist student progress.
  - Some elementary schools are developing their own diagnostic assessments for the different strands in mathematics. The value of developing a comprehensive approach to diagnostic assessment tools at the provincial level was discussed.
  - Participants spoke about the power of assessment "for" and "as" learning as a way to garner evidence of student mathematical thinking which is so crucial for effective mathematics instruction.
  - Involving students in the assessment process was considered important as was providing ongoing feedback, sharing learning goals and jointly constructing success criteria.

6. Effective instruction builds on students' prior knowledge and understanding.

- Pedagogical documentation – using notes, slides, videos and so on to show student thinking and learning – was discussed as an innovative way to assess student readiness to learn new concepts in mathematics and to identify next steps in learning and improvement processes.

- As teachers plan next steps in mathematics learning, they can use the “artifacts” of documentation to anticipate student responses and to reflect on what they see or hear in student answers.
- Keeping a log of student learning provides useful assessment information and can help inform efforts to personalize and differentiate learning.

7. Greater alignment is needed between elementary and secondary schools.

- Understanding the continuum of learning from elementary programming in Grades 7 and 8 to secondary programming in Grades 9 and 10 is vital for improving student outcomes in mathematics.
- Time for Grades 7–12 teachers of mathematics to learn together was seen as important, as these years bring special challenges for students.
- Opportunities for teachers Grades 7–12 to learn together and share successful practices will help develop a common understanding of the continuum of learning in the mathematics curriculum.

8. There is no single strategy to improve mathematics teaching and learning.

- Teachers who know how to infuse variety and challenge into lessons, and how to pace them, can help build student confidence and success.
- Student interest is promoted through authentic tasks which may be integrated across mathematics strands and/or subject areas.
- Students benefit from opportunities to develop procedural fluency and automaticity. A variety of instructional methods can be applied for acquiring these skills.
- Using math warm-ups, chunking of time within a lesson and careful sequencing of concepts as well as many opportunities to revisit them throughout the course were highlighted as important strategies.
- Providing ongoing feedback and checking in for understanding supports student success.
- Manipulatives and technology were highlighted as useful instructional tools.
- Board representatives spoke of the impact skillful teaching has on students in Grade 9 Applied Mathematics when a positive learning environment is nurtured.

9. Direct instruction and practice have their place in a mathematics program.

- Direct teaching and practice of specific skills have a definite place – aving a deep understanding of the curriculum expectations helps teachers determine when direct instruction may be needed.

- Some students require more direct instruction than others. Personalization of learning requires educators to be attuned to which students need more assistance and which students can work more independently.
- When students are given opportunities to revisit mathematical concepts throughout a course of study or term, they develop deeper understanding. A cyclical approach in conjunction with explicit and differentiated instruction appears to be effective.
- Students would benefit by teachers explicitly developing cross-curricular lessons that link the learning expectations in mathematics not only to literacy but to all subject areas (e.g., science, history, art).

10. The visual and tactile representation of mathematical procedures and processes is useful for promoting deeper learning.

- The use of manipulatives needs to be encouraged in all classrooms, K–12.
- Researchers spoke about the utility of manipulatives (both concrete and virtual) to help students move from concrete to abstract thinking as well as from abstract to concrete as a way to develop and deepen conceptual understanding.
- The notion of a continuum (concrete to abstract; abstract to concrete) was discussed as representing a cultural shift for many schools – multiple instructional strategies are needed in classrooms, especially in secondary schools where the use of manipulatives is less common.

SOME OF THE OUTSTANDING CHALLENGES

How do we deepen the expertise of teachers in mathematics in our elementary schools?

- Increased math content knowledge was discussed as key to further progress.
- In some boards, principals and vice-principals are the math leads – they, too, could benefit from opportunities to deepen their expertise with math content and their understanding of the math behind the approaches and strategies that need to be in place in their schools.
- Professional learning for coaches and facilitators was discussed as very important as well. Board representatives spoke about the benefit of using skilled facilitators with deep math content knowledge to help support growth in mathematics.
- Teacher education programs would benefit by increasing the amount of math prospective teachers need to know. Discussion explored the benefits that the two-year teacher education program might have in this regard.
- Opportunities for job-embedded professional learning using samples of actual student work (as discussed above – see point 6) was viewed as vital by many participants. On-site coaching, particularly by a content expert and skilled facilitator, was considered essential to improvement, though acknowledged as time-intensive work.

How can we build common understandings of effective practices across schools?

- Participants suggested that in contrast to the implementation of literacy improvement strategies, mathematics improvement strategies are at an earlier stage.
- They spoke about the challenge in developing common understandings of effective mathematics pedagogy across classrooms, schools and districts. The sheer size of some boards makes consistent implementation a significant challenge. Making widespread improvements – what some researchers call "going to scale with system learning" – was a common theme.
- Many different roles are involved in improving mathematics education – superintendents, principals, consultants, Student Success teachers and Student Work Study Teachers, for example. Aligning the work of those charged with supporting improvement was discussed as important and challenging.

How can we better inform parents about how their children are learning mathematics?

- Many parents would benefit by understanding the goals of the Mathematics curriculum and how math is learned effectively. Parents may not understand why their children are not being taught as they themselves were taught.
- Clear communication is required to help parents understand the need for their child to develop both conceptual understanding and procedural fluency and how a problem-posing and problem-solving learning environment can develop both mathematical thinking and computational skills.

What role might technology play in improving the instruction of mathematics?

- While technology is being used to build teacher capacity in some boards, it is still unclear to many as to what exact role technology might play.
- Several important questions were raised: Are we using technology effectively in the teaching of mathematics? Which technologies or technological programs should be in every school?
- How do we address the equity questions that technology raises in terms of access and use?

Key Considerations for Improvement

The Forum for Action was a rich learning opportunity for all involved, fostering much interest in further dialogue and some common ground as well about the most promising next steps for Ontario mathematics education. These points of convergence have been culled from researcher presentations and school and board discussions, as follows (in no order of importance):

1. We need to continue to work together – the Ministry of Education, school districts, faculties of education and teachers –in acquiring deeper content knowledge of mathematics and improved pedagogical knowledge and skill.

2. Increasing teachers' deep content knowledge in conjunction with their pedagogical knowledge will require knowledgeable and skillful facilitators working with them -- hands-on – in school and classroom settings.

3. School districts require multiple approaches to sharing promising practices – both within and across schools – for intervening with students who are struggling in mathematics.

4. A team approach – where teachers plan, learn and teach together – is vital for improving mathematics teaching and learning on a wide scale.

5. Teaching mathematics effectively requires a range of strategies, including the purposeful design of lessons, a careful sequencing of student activities and appropriate interventions and scaffolds for student learning. It also requires knowing students well to work with the strengths, needs and interests they bring to school.

6. Improving mathematics outcomes requires building the confidence of both teachers and students in their ability to go deeper into learning mathematics.

7. Spatial reasoning is an important predictor among early learners of later success in mathematics – visual representations are essential for developing spatial reasoning and supporting growth in student understanding of mathematical concepts.

8. Manipulatives that help illustrate mathematical functions and computations are excellent and appropriate tools for all student learning K–12 and for teacher and administrator learning as well. When manipulatives are used regularly, they help learners move from a concrete understanding to a more abstract one and back again.

9. Many students with learning difficulties benefit especially from sustained use of manipulatives along with precise instruction that meets their individual learning needs.

10. An area for continued research and development is the use of technology in mathematics. Technology can be helpful for many students who have specific learning difficulties and its use for all students needs to be explored.

11. Quality early learning in mathematics is essential as a foundation. We need to become systematic about quality early years experiences in mathematics.

12. Staffing is an important variable and strategic staffing – placing the highest skilled teachers where needs are greatest – has proven to be valuable in improvement efforts.

13. Mathematics programming needs to involve time for the development of procedural fluency complemented by building strong conceptual understandings.

References

Brochu, P., Gluszynski, T., & Cartwright, F. (2011). *Second report from the 2009 Programme for International Student Assessment.* Toronto: Council of Ministers of Education, Canada (CMEC).

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., & Japel, C. (2007). *School readiness and later achievement. Developmental psychology, 43(6)*, 1428.

Fisher, K., Hirsh-Pasek, K .,Golinkoff, R. M., Singer, D., & Berk, L. E. (2010). Playing around in school: Implications for learning and educational policy. In A. Pellegrini (Ed.), *The Oxford handbook of play.* NY: Oxford University Press, 341–363.

Heckman, J. J. (2006). Skill formation and the economics of investing in disadvantaged children. *Science, 312* (June). www.sciencemag.org

Mourshed, M., Chijioke, C. & Barber, M. (2010). *How the world's most improved school systems keep getting better.* http://mckinseyonsociety.com/

OECD. (2010). PISA 2009 Results: What students know and can do – Student performance in reading, mathematics and science (Vol. I). http://www.oecd.org/pisa/pisaproducts/48852548.pdf

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35*(3), 3–14.

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, *139*(2), 352–402.

Verdine,B N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2013). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development*, 1–14.