Instead of telling students what to do ...

“Never say anything a kid can say! This one goal keeps me focused. Although I do not think that I have ever met this goal completely in any one day or even in a given class period, it has forced me to develop and improve my questioning skills. It also sends a message to students that their participation is essential. Every time I am tempted to tell students something, I try to ask a question instead.”

(Reinhart, 2000, p. 480)

Asking Effective Questions

Provoking student thinking/deepening conceptual understanding in the mathematics classroom

Researchers support a problem-solving approach in the mathematics classroom because it engages students in inquiry, prompting them to build on and improve their current knowledge as they “construct” explanations that help them solve the task at hand. “In a constructivist classroom,” Marian Small writes, “students are recognized as the ones who are actively creating their own knowledge” (2008, p. 3). The teacher’s skilful questioning plays a vital role in this context, helping students to identify thinking processes, to see the connections between ideas and to build new understanding as they work their way to a solution that makes sense to them.

In order to know what questions to ask to move the mathematical ideas forward, it is critical that teachers continually work to develop their knowledge of mathematics-for-teaching as they connect this understanding to the curriculum. By listening attentively to students’ ideas and keeping the learning goal and the big mathematical ideas in mind, we are able to identify and develop the important ideas in the students’ discourse.

In addition to making decisions about what questions to ask during student discussions, teachers can plan effective questions to ask as they prepare lessons. Knowing the development of big ideas across the curriculum, reading material in teacher resources and solving problems themselves are examples of activities that may support teachers as they determine which questions to ask during lessons.

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The classroom becomes a workshop ...
“... as learners investigate together. It becomes a mini-society — a community of learners engaged in mathematical activity, discourse and reflection. Learners must be given the opportunity to act as mathematicians by allowing, supporting and challenging their ‘mathematizing’ of particular situations. The community provides an environment in which individual mathematical ideas can be expressed and tested against others’ ideas. ... This enables learners to become clearer and more confident about what they know and understand.”

(Fosnot, 2005. p. 10)

1. **ANTICIPATE STUDENT THINKING**
An important part of planning a lesson is engaging in solving the lesson problem in a variety of ways. This enables teachers to anticipate student thinking and the multiple ways they will devise to solve the problem. This also enables teachers to anticipate and plan the possible questions they may ask to stimulate thinking and deepen student understanding.

2. **LINK TO LEARNING GOALS**
Learning goals stem from curriculum expectations. Overall expectations (or a cluster of specific expectations) inform teachers about the questions to ask and the problems to pose. By asking questions that connect back to the curriculum, the teacher helps students centre on these key principles. During the consolidation phase of the three-part lesson (see pages 7 and 8), students are then better able to make generalizations and to apply their learning to new problems.

**Linking to Learning Goals**
Example for the big idea *The same object can be described by using different measurements.*
Teacher’s learning goal: To make a connection between length, width, area and multiplication.
Problem question: A rectangle has an area of 36 cm². Draw the possible rectangles.
Possible questions:
- As you consider the shapes you made, what are the connections of the length of the sides to the total area?
- If you know the shape is a rectangle, and you know the total area and the length of one side, what ways can you think of to figure out the length of the other three sides?

3. **POSE OPEN QUESTIONS**
Effective questions provide a manageable challenge to students – one that is at their stage of development. Generally, open questions are effective in supporting learning. An open question is one that encourages a variety of approaches and responses.
Consider “What is 4 + 6?” (closed question) versus “Is there another way to make 10?” (open question) or “How many sides does a quadrilateral figure have?” (closed question) versus “What do you notice about these figures?” (open question). Open questions help teachers build student self-confidence as they allow learners to respond at their own stage of development. Open questions intrinsically allow for differentiation. Responses will reveal individual differences, which may be due to different levels of understanding or readiness, the strategies to which the students have been exposed and how each student approaches problems in general. Open questions signal to students that a range of responses are expected and, more importantly, valued. By contrast,
yes/no questions tend to stunt communication and do not provide us with useful information. A student may respond correctly but without understanding.

Invitational stems that use plural forms and exploratory language invite reflection. Huinker and Freckman (2004, p. 256) suggest the following examples:

As you think about... As you consider...
Given what you know about... In what ways...
In regard to the decisions you made... In your planning...
From previous work with students... Take a minute...
When you think about...

4. **POSE QUESTIONS THAT ACTUALLY NEED TO BE ANSWERED**
Rhetorical questions such as “Doesn’t a square have four sides?” provide students with an answer without allowing them to engage in their own reasoning.

5. **INCORPORATE VERBS THAT ELICIT HIGHER LEVELS OF BLOOM’S TAXONOMY**
Verbs such as connect, elaborate, evaluate and justify prompt students to communicate their thinking and understanding, to deepen their understanding and to extend their learning. Huinker and Freckman (2004, p. 256) provide a list of verbs that elicit specific cognitive processes to engage thinking:

- observe, evaluate
- notice, summarize
- remember, visualize (“see”)
- contrast, differ
- interpret, distinguish
- decide, conclude
- identify, infer
- compare, relate
- predict, consider
- explain, describe

6. **POSE QUESTIONS THAT OPEN UP THE CONVERSATION TO INCLUDE OTHERS**
The way in which questions are phrased will open up the problem to the big ideas under study. The teacher asks questions that will lead to group or class discussions about how the solution relates to prior and new learning. Mathematical conversations then occur not only between the teacher and the student, but also between students within the classroom learning community.

7. **KEEP QUESTIONS NEUTRAL**
Qualifiers such as easy or hard can shut down learning in students. Some students are fearful of difficult questions; others are unchallenged and bored by easy questions. Teachers should also be careful about giving verbal and non-verbal clues. Facial expressions, gestures and tone of voice can send signals, which could stop students from thinking through.

8. **PROVIDE WAIT TIME**
When teachers allow for a wait time of three seconds or more after a question, there is generally a greater quantity and quality of student responses. When teachers provide wait time, they find that less confident students will respond more often; many students simply need more time than is typically given to formulate their thoughts into words. Strategies like turn and talk, think-pair-share and round robin give students time to clarify and articulate their thinking. (For strategies to maximize wait time, See *A Guide to Effective Literacy Instruction, Grades 4 to 6 – Volume 1* (Part 2, Appendix). The Guide offers tips for using these strategies in the “Listening and Learning from my Peers” section on page 134.)

(This tip list has been drawn from Baroody, 1998, pp. 17-18. See also *A guide to effective instruction in mathematics, Kindergarten to Grade 6 – Volume Two: Problem solving and communication*, pp. 32-33.)
Stimulate thinking by asking open-ended questions ...

- How else could you have...?
- How are these _____________ the same?
- How are these different?
- About how long...? (many, tall, wide, heavy, big, more, less, etc.)
- What would you do if...?
- What would happen if...?
- What else could you have done?
- If I do this, what will happen?
- Is there any other way you could...?
- Why did you...?
- How did you...?

TO HELP STUDENTS SHARE THEIR REPRESENTATIONS
(and show/describe/demonstrate/represent)
Questions to pose:
- How have you shown your thinking (e.g., picture, model, number, sentence)?
- Which way (e.g., picture, model, number, sentence) best shows what you know?
- How have you used math words to describe your experience?
- How did you show it?
- How would you explain ____________ to a student in Grade ___? (a grade lower than the one the student is in)

Prompts to use:
- I decided to use a...
- A graph (table, T-chart, picture) shows this the best because...
- I could make this clearer by using a...
- The math words that help someone understand what I did are...

TO HELP STUDENTS REFLECT ON THEIR WORK
(and analyze/compare/contrast/test/survey/classify/sort/show/use/apply/model)
Questions to pose:
- What mathematics were you investigating?
- What questions arose as you worked?
- What were you thinking when you made decisions or selected strategies to solve the problem?
- What changes did you have to make to solve the problem?
- What was the most challenging part of the task? And why?
- How do you know?
- How does knowing __________ help you to answer the questions __________?

Prompts to use:
- A question I had was...
- I was feeling really...
- I decided to ____________, I was thinking...
- I found ____________ challenging because...
- The most important thing I learned in math today is...

TO HELP STUDENTS MAKE CONNECTIONS
(and connect/relate/refer/imagine/describe/compare)
Questions to pose:
- What does this make you think of?
- What other math can you connect with this?
- When do you use this math at home? At school? In other places?
- Where do you see ____________ at school? At home? Outside?
- How is this like something you have done before?

Prompts to use:
- This new math idea is like...
- I thought of...
- I did something like this before when...
- We do this at home when we...
- I remember when we...
Prompts to Get Students Thinking

**To Help Students Share Their Feelings, Attitudes or Beliefs about Mathematics**
(and share / reflect / describe / compare / tell)
Questions to pose:
- What else would you like to find out about ___________?
- How do you feel about mathematics?
- How do you feel about ___________?
- What does the math remind you of?
- How can you describe math?
Prompts to use:
- The thing I like best about mathematics is ...
- The hardest part of this unit on ___________ is ...
- I need help with ___________ because ...
- Write to tell a friend how you feel about what we are doing in mathematics.
- Mathematics is like ___________ because ...
- Today, I felt ...

**To Help Students Retell**
(and tell / list / recite / name / find / describe / explain / illustrate / summarize)
Questions to pose:
- How did you solve the problem?
- What did you do?
- What strategy did you use?
- What math words did you use or learn?
- What were the steps involved?
- What did you learn today?
- What do(es) ___________ mean to you?
Prompts to use:
- I solved the problem by ...
- The math words I used were ...
- The steps I followed were ...
- My strategy was successful because ...
- Explain to a young child or someone that wasn’t involved ...
- Draw a picture to show how you solved the problem.

**To Help Students Predict, Invent or Problem Solve**
(and create / plan / design / predict / imagine / devise / decide / defend / solve / debate)
Questions to pose:
- What would happen if ...?
- What decisions can you make from the pattern that you discovered?
- How else might you have solved the problem?
- Will it be the same if we use different numbers?
- What things in the classroom have these same shapes?
- How is this pattern like addition?
- What would you measure it with? Why?
- How are adding and multiplying the same?
Prompts to use:
- Prove that there is only one possible answer to this problem.
- Convince me!
- Tell me what is the same? What is different?
- How do you know?

Stimulate thinking by asking open-ended questions ...
- How do you know?
- What does (this) ___________ represent?
- How did you know where ...?
- How did you know which ...?
- How did you know when ...?
- Could you use some other materials to ...?
- How could you record your work?
- How could you record your discovery?
- How could you share your discovery?
- How did you estimate what the answer could be?
- How did you prove your estimate?
When are we telling students too much?

Our goal in posing questions is not to lead our students to a pre-determined solution. Rather, our goal is to help students clearly identify their thinking about the problem. Our questions will help students see the connections between ideas as they naturally make sense of mathematical concepts. Although students can gather some information through the process of their own reasoning, there will be some information that they can’t draw from logic. For example, a student can derive a procedure such as adding two numbers (a student-generated algorithm), but they will not be able to come up with the name of a figure that has four sides.

James Hiebert (2003) suggests three situations in which teachers might consider conveying information to students:

1. Students need conventional written notations. For example, how to represent a fraction, how to show that a quantity is greater than another and terms related to solving problems that require them. For example, “You’re saying this triangle has no equal sides. You’re describing a scalene triangle.” Students don’t always have the language to respond to open questions such as, “How do you know?” Modelling this language is important in building students’ self-efficacy.

2. During consolidation, teachers may present alternative methods that have not been suggested by students. Teachers may choose to do this if a particular strategy would help students better understand the big idea underlying the problem. The strategy should be presented as just another alternative, and not as the preferred strategy.

3. Again during consolidation, teachers may highlight the mathematical ideas embedded in the students’ solutions. These can be made explicit by posing questions that focus the students’ attention on these ideas. The teacher may annotate the solutions to make these ideas visible, and add them to the chart of summary and highlights constructed by the class. For example, the teacher may record right on a student solution that the authors have used the strategy of making friendly numbers in order to solve the problem. Another teacher might highlight the way the students have shown how multiplication and division are related.

Supporting Student Inquiry in the Problem-Solving Lesson

Each part of the problem-solving lesson has a different purpose. Consequently, the purpose of the questions and prompts during each part will vary. A teacher’s knowledge of the big ideas and concepts in the curriculum has an impact on which questions to ask and when to ask them.

1. BEFORE – ACTIVATION, MINDS-ON

The purpose of the first part of the lesson is to get the students cognitively prepared for the lesson problem by having them think about ideas and strategies that they have previously learned and used. The teacher might also ask students to solve a smaller problem in order to evoke prior knowledge and familiar skills and strategies.

Through questioning, teachers seek to establish what students already know about the content and the context of the problem. Teachers listen for an indication of students’ level of readiness. Careful observation during this stage will allow teachers to pose further questions and prompts that provoke deeper thinking throughout the lesson. It is during the activation phase that students will recall previous learning and be prepared to apply it to the lesson problem and discussion. This is a good opportunity for students to ask questions of each other that are similar to those modelled by their teachers.
Below are some examples of prompts and questions that may be posed by teachers and/or by students during the activation phase; depending on the context, they may also be appropriate during other parts of the lesson. It is helpful to consider the purpose of the question or prompt.

Possible purpose: Understand the mathematical task and its context
- What information are you/we going to use to solve this problem?

Possible purpose: Establish connections, Activate prior knowledge
- What other problem have you/we solved that is similar to this one?
- Let’s record all that you/we know about ________.
- What do these two ______ have in common? What is unique about each of them?

2. During – Working on it, Action
During this part of the lesson, students are working on solving a problem and communicating and representing their mathematical thinking. This phase of the lesson provides multiple observational assessment opportunities, which may be captured as anecdotal or digital records.

As students work to make sense of the mathematical ideas embedded in the problem, both teachers and students use questions to develop and clarify their mathematical thinking. Solving the problem during the planning phase helps teachers to anticipate some of the challenges that students may encounter and informs their questioning. By listening closely to students as they discuss their emerging solutions, teachers occasionally pose questions and use the information they glean to inform their in-the-moment instructional decisions. This information may also be used as assessment-for-learning data for planning purposes, including the planning of questions for lesson consolidation. Both correct and incorrect solutions can be probed, since questioning promotes the thinking necessary to build, construct and consolidate understanding. At times, however, questioning may not be appropriate, since the students need time to persevere through their thinking without interruption.

Below are examples of prompts and questions that teachers and students may pose in order to probe and provoke mathematical thinking. In modelling and charting good questioning, teachers equip students to be able to ask good questions of each other.

Possible purpose: Clarify reasoning and understanding
- How do you/we know?
- What does this part represent in your/our solution?
- How do you know your/our answer is reasonable?

Possible purpose: Focus on communication
- What could you/we add to your/our solution to make it clearer for the reader?
- How can you/we represent your/our thinking?

Possible purpose: Make a conjecture
- What would happen if...?
- Would this work every time? Can you/we think of any examples that don’t work?

3. After – Consolidation, Highlights and Summary, and Practice
In this phase of the three-part lesson, the teacher strategically coordinates student sharing of their solutions to the lesson problem, using a mathematical instructional strategy like bansho, math congress or a gallery walk. During consolidation, teachers and students ask questions that help to summarize the mathematical ideas embedded in the class solutions. They support students in establishing explicit connections.
between solutions, concepts and strategies. As students analyze other students’ solutions, they question their own ideas and the ideas of others. They examine mathematical thinking, engage in metacognition and make generalizations related to the learning goal. By seeing other ways to solve the problem, they may adopt new strategies when they solve subsequent problems.

Below are some examples of prompts and questions teachers and students may pose during the consolidation phase.

**Possible purpose: Clarify strategies and understanding**
- How could you/we arrive at the same answer but in a different way?
- How would you/we explain what ________ just said, in your/our own words?
- Convince them/me/us!

**Possible purpose: Reason mathematically**
- Have you/we found all the possibilities? How do you/we know?

**Possible purpose: Compare solutions and look for commonalities**
- How is this solution similar to or different from the others?

**Possible purpose: Synthesize learning, articulate generalizations**
- What have you/we discovered about ________ while solving this problem?
- What have you/we learned today?

**In Summary**

Questioning is a powerful instructional strategy. Open questions that are related to the big ideas embedded in the curriculum expectations and learning goals will excite student curiosity, provoke critical thinking, elicit reflection and help students construct their own meaning for the mathematics they are studying. Their responses will help the teacher assess what students know and what next instructional steps might be. Developing skills in questioning for understanding and content knowledge evolves over time and like anything else, requires practice. The payoff is significant in terms of students’ conceptual understanding.

**References**

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