

Webcast Professional Learning Series
Student Achievement Division

Viewer's Guide

Understanding Geometric Figures Through Drawing and Paper Folding

**Featuring Dr. Akihiko Takahashi,
DePaul University, Chicago**

Multi-media resource for professional learning



reach every student





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On this DVD you will find a Print and Video Resources folder which contains WMV files for PowerPoint presentations, this Viewer's Guide (PDF) and Organizer #1 (Key Ideas).

To order the multi-media package
Understanding Geometric Figures Through Drawing and Paper Folding

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The webcast segments and related resources are also accessible online at
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You may wish to pace your mathematical study as follows:

- Pause and play each segment.
- Engage in the mathematical work and discussion prompted throughout this webcast.
- Keep a record of your mathematical thinking and the teaching strategies demonstrated throughout this webcast.
- Between study sessions, try the activities with students.
- At the following session, analyse artefacts (student work, video clips) to consider questions and feedback that might move the students forward in their learning.

Overview

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Geometry helps us represent and describe objects and their interrelationships in space. Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions.

Ontario Ministry of Education, 2005

This webcast explores the study of 2-D geometry in the junior mathematics classroom. It offers glimpses into the acquisition of new learning by educators who worked with Dr. Akihiko Takahashi, an internationally acclaimed expert in Japanese lesson study, and it identifies some powerful strategies for increasing student understanding of the concepts and language of geometry.

The mathematics study undertaken by educators and students in this video relates to the expectations of the Ontario curriculum. Specifically, *Understanding Geometric Figures Through Drawing and Paper Folding*:

- demonstrates strategies such as paper folding to construct conceptual understanding of geometric terminology
- shows how different material such as compasses, protractors, set squares, rulers and straws of different lengths can be used to construct a conceptual understanding of geometry
- provides a series of activities along a trajectory linking to the van Hiele taxonomy of geometric thinking

The goals of this webcast include:

- deepening the understanding of geometric thinking and the properties and relationships of 2-D shapes
- experiencing the use of set squares, compasses, protractors and paper folding to investigate the properties of 2-D shapes
- becoming more familiar with possible student responses to geometry tasks

Students' Development of Geometric Thinking

All the geometric concepts explored in this webcast relate to the mathematics expectations of the Ontario curriculum.

- Grade 1 – relate shapes to other shapes.
- Grade 2 – distinguish between attributes that are geometric properties and attributes that are not; identify and describe various polygons (i.e., triangles, quadrilaterals) and sort and classify them by their geometric properties (number of sides and vertices) using concrete materials and pictorial representations; compose and decompose two-dimensional shapes.
- Grade 3 – use a reference tool to identify right angles and to describe angles as greater than, equal to, or less than a right angle; identify and compare various polygons (triangles, quadrilaterals) and sort them by their geometric properties (i.e., number of sides, side lengths, number of interior angles, number of right angles); explain the relationships between different types of quadrilaterals (e.g., a square is a rectangle because a square has four sides and four right angles).
- Grade 4 – identify and compare different types of quadrilaterals (rectangle, square) and sort and classify them by their geometric properties (e.g., sides of equal length, parallel sides, number of right angles); identify benchmark angles (i.e., straight angle, a right angle, and half a right angle) using a reference tool and compare other angles to these benchmarks; relate the names of benchmark angles to their measures in degrees (e.g., a right angle is 90°).
- Grade 5 – identify and classify acute, right, obtuse, and straight angles; measuring angles to 90° with a protractor; identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral) and classify them according to angle and side properties; construct triangles using a variety of tools (e.g., protractor, compass, dynamic geometry software), given acute or right angles and side measurements.
- Grade 6 – sort and classify quadrilaterals by their geometric properties related to angles and sides; measure angles to 180° with a protractor.

Excerpted from *The Ontario Curriculum Grades 1–8: Mathematics*, 2005 (revised)

From Viewing to Action

Ideas to ponder ...

- How do the ideas and strategies illustrated in this webcast relate to your own teaching context?
- How might you identify the strategies you are currently using to support student learning?
- Which new strategies will you add to your instructional repertoire – in both the short term and the long term?
- What benefits might there be to having a team approach to learning?

Key Ideas Explored in This Resource

A lesson without learners having the opportunity to express generality is not a mathematics lesson.

Mason, Graham & Johnson-Wilder, 2005

The processes of reasoning and generalizing are fundamental to learning mathematics. As students revisit conjectures that they have found to be true in one context to see if they are always true, they are in the process of generalizing.

The reasoning process supports a deeper understanding of mathematics by enabling students to make sense of the mathematics they are learning. The process involves exploring phenomena, developing ideas, making mathematical conjectures and justifying results.

Teachers draw on students' natural ability to reason to help them learn to reason mathematically. Initially, students may rely on the viewpoints of others to justify a choice or an approach. Students should be encouraged to reason from the evidence they find in their explorations and investigations, or from what they already know to be true, and to recognize the characteristics of an acceptable argument in the mathematics classroom.

As you view this resource, you may wish to record your thinking about how reasoning and generalizing are essential to the learning of mathematics.

Organizer #1 - Key Ideas

<i>Key Idea</i>	<i>What do you notice?</i>	<i>How might this apply to your practice?</i>	<i>What wonderings do you have about this idea?</i>
<i>Reasoning</i>			
<i>Generalizing</i>			

Organizer #1 is available in Word and in PDF in the Print Resources folder on the DVD.

WEBCAST SEGMENTS

Introduction

(2:35)

Dr. Takahashi invites us to consider how educators design instruction in geometry and spatial sense that:

- builds on students' everyday knowledge and previous mathematical knowledge
- goes beyond vocabulary to concepts
- builds from one lesson to include not just the facts, but also the strategies and habits of thinking

Discussion:

- *How do the ideas of noticing students' mathematical thinking relate to teaching and learning?*
- *What are your students' understandings about geometry? How do you know?*
- *What do you wonder about the learning of geometry?*

Research About Students' Development of Geometric Thinking

(3:24)

This clip shares some of the van Hiele research about the development of geometric thinking . (See pages 8 and 9 for an overview of the van Hiele taxonomy.)

Discussion:

- *In examining the Ontario mathematics curriculum, what connections do you see with the van Hiele research regarding the learning trajectory in geometric thinking?*
- *In considering the mathematical processes of problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing and communicating, what connections do you see with the van Hiele phases of experiences?*

The van Hiele Taxonomy of Geometric Thinking

Progress from one level to the next level is more dependent on educational experiences than on age or maturation. For geometry learning in Ontario's elementary curriculum, Level 0, Level 1 and Level 2 are relevant.

Level 0 » Visualization

Students focus on what individual shapes “look like.”

Level 1 » Analysis

Students conceive shapes as being part of a group of similar shapes and begin to notice their properties. For example, a student may notice that having opposite sides parallel is a property of a parallelogram.

Level 2 » Informal Deduction

Students have greater capacity to apply “If ... then ...” reasoning and can consider simple logical arguments about properties of 2-D shapes. For example, a student can articulate that a square is a type of rectangle because a square has all the properties of a rectangle.

Level 3 » Deduction

Students understand the interrelationship and roles of undefined terms, axioms, definitions, theorems and proofs. Students construct proofs and can articulate statements and their converse. Of note, *The Ontario Curriculum, Grades 9 and 10:Mathematics*, 2005 (revised) and *Grades 11 and 12:Mathematics*, 2007 (revised) include a small number of expectations related to the van Hiele's third level of geometric thinking.

Level 4 » Rigour

Students work within and across a variety of geometry systems in post-secondary education.

Note: The use of the word “level” in the van Hiele taxonomy is different from the use of “level” in the Ontario Achievement Chart.

Experiences that Augment Geometric Thinking within Each van Hiele Level

Some experiences can facilitate (or impede) learning within a level. A student may need to cycle through some of the five phases more than once within a particular topic.

Information » Through discussion, the teacher identifies what students already know about a topic and the students become oriented to the new topic.

Guided Orientation » Students explore the objects of instruction in carefully structured tasks such as folding, measuring or constructing. The teacher ensures that students explore specific concepts.

Explication » Students describe what they have learned about the topic in their own words. The teacher introduces relevant mathematical terms.

Free Orientation » Students apply the relationships they are learning to solve problems and investigate more open-ended tasks.

Integration » Students summarize and integrate what they have learned, developing a new network of objects and relations.

The van Hieles assert that progress through the levels is more dependent on the instruction received than on age or maturation. Thus, the method and organization of instruction, as well as the content and materials used, are important areas of pedagogical concern.

M.L. Crowley, 1987

Educators in Mathematical Study

IN BRIEF

(9:46)

Ontario mathematics educators, working with Dr. Takahashi, brainstorm the geometric concepts, actions and strategies that elementary students learn from Kindergarten to Grade 6. They explore ways that students can effectively learn about geometry within the classroom.

Discussion:

Dr. Takahashi suggests the following:

- *When we teach something new, we need to build on students' prior knowledge.*
- *We shouldn't tell students an answer is 'right' or 'wrong,' but rather we should cultivate reasoning skills through these activities.*
- *Dr. Takahashi offers a number of other ideas. Which ones offer new insight into the learning of geometry for you? In what way?*

IN DEPTH

As you view each clip, consider:

- What are the geometric actions, concepts and relationships being shown?
- How is each activity linked to the one before it and to the one after it?
- What else do you notice? How might it support students' learning?

Analysing Triangles and Quadrilaterals

Drawing Line Segments to Construct 2-D Shapes

(6:37)

Dr. Akihiko Takahashi discusses the importance of activating students' previous knowledge and experience with triangles and quadrilaterals for new learning. Discussion with the study group ensues about identifying and connecting points in relation to drawing line segments, quadrilaterals and triangles. The mathematical actions taken and the activity context for drawing quadrilaterals and triangles are significantly related to their geometric definitions.

Discussion:

- *Compare your definition of quadrilaterals and triangles to the conceptual descriptions presented. How are they similar? How are they different?*

Examining Triangles and Quadrilaterals Using Sides and Vertices

(8:53)

Akihiko explains that distinguishing between triangles and quadrilaterals requires analysis of their components in terms of the number of sides and vertices. The study group suggests several classroom strategies and possible student learning difficulties.

Discussion:

- *What developmental sequence of learning activity does Akihiko recommend for the concepts of quadrilaterals and triangles?*
- *How can the activities, "Let's find the triangles and quadrilaterals" and "Connect dots to make triangles and quadrilaterals" be used to develop students' conceptual understanding of triangles and quadrilaterals?*
- *What strategies could students use to distinguish quadrilaterals and triangles? What difficulties might students have? Explain.*

Constructing Angles and 2-D Shapes Through Paper Folding

Constructing a Right Angle Through Paper Folding

(6:00)

What does it mean to have a right angle? Akihiko and the study group examine the use of paper folding as a strategy to determine the meaning of a right angle. The Grade 4 and 5 students use set squares and paper folding models to determine right angles.

Discussion:

- *How does paper folding model the concept of a right angle? What difficulties could students encounter when paper folding?*
- *How can paper folding from irregular sized paper and the use of set squares help students to understand the concept of a right angle?*

Constructing a Rectangle Through Paper Folding

(1:52)

Akihiko and the study group continue to fold paper to construct the four right angles of a rectangle. The Grade 4 and 5 students measure the lengths of a rectangle to notice geometric relationships between the sides of a rectangle.

Discussion:

- *Why is it that folding paper to make right angles 4 times makes a rectangle? Will this always happen? Why do you think this?*
- *Why is it important to have students measure the side lengths of a rectangle in order to discern geometric relationships?*

Constructing a Square Through Paper Folding

(2:15)

The Grade 4 and 5 students and the study group use paper folding to construct a square and discuss strategies for proving that the construction is a square.

Discussion:

- *How does paper folding model the geometric properties of a square?*
- *How does paper folding compare to the way(s) that you have learned about the properties of a square?*

Analysing Geometric Relationships Between 2-D Shapes and Properties of 2-D Shapes

Analysing the Relationship Between Rectangles and Squares

(2:10)

Akihiko and the study group discuss the geometric properties of rectangles and squares and an organizational hierarchy for quadrilaterals is partially represented.

Discussion:

- *How are rectangles and squares similar? How are they different? How can you prove it using a paper-folding model?*
- *Why is it important to consistently refer back to the original definition of a rectangle when comparing rectangles and squares?*

Teachers have a critical role to play in ensuring that tools are used effectively to support students to organize their mathematical reasoning and support their sense-making.

Blanton & Kaput, 2005

Exploring the Relationship Between a Rectangle (Square) and a Right Triangle (3:08)

Akihiko and the study group use paper folding and cutting to construct a triangle from a square and rectangle. The Grade 4 and 5 students use their set squares to determine right triangles.

Discussion:

- Describe the relationship between a square and a triangle. How does paper folding help to model this geometric relationship?
- What is the difference between the triangles constructed from a rectangle and a square? Why is this so?

Investigating the Properties and Types of Triangles (7:38)

Akihiko and the study group examine a geometry task developed from a lesson study that challenges students to make different triangles from multiple copies of four different straw lengths. Grade 4 and 5 students describe and name the triangles they make.

Discussion:

- Can a triangle always be made with any combination of straw lengths: 6 cm, 8 cm, 10 cm and 12 cm? How do you know?
- How could you use this triangle problem within a three-part problem solving lesson (Before, During, After – Consolidation, Highlights/Summary, Practice) so that students are learning the properties of different triangles?
- Dr. Takahashi shares how in lesson study, the teachers who developed this activity specifically used four different straw lengths. When have you experienced the importance of small details in creating effective learning situations? In what way?

Using Geometric Tools – Set Squares, Protractor and Compass

Making 2-D Shapes with Set Squares

(5:00)

The study group and the Grade 4 and 5 students use set squares to compose quadrilaterals and triangles. The study group discusses explanations about why particular set squares create particular 2-D shapes using the geometric properties of the 2-D shapes of the set squares, such as inside or interior angles.

Discussion:

- *What mathematical actions did the students take to make 2-D shapes using set squares?*
- *List angle problems that could be posed combining set squares.*

Measuring Angles

(0:51)

The difference between the measurement of an angle and the concept of an angle is discussed.

Discussion:

- *How do Ontario curriculum expectations articulate the development of the concept and measurement of an angle?*
- *What geometry task or problem could be provided to students so that they can understand the difference between the **measurement** and the **concept** of an angle?*

Constructing Triangles Using a Compass

(3:09)

The study group and the Grade 4 and 5 students explore the use of the compass and ruler to construct triangles of specific lengths. The study group discusses challenges that students might have when constructing triangles using a compass.

Discussion:

- *How is the distance of the compass arm related to the construction of a triangle of particular side lengths?*
- *What questions could you pose in response to student's difficulties in using the compass and ruler to construct a triangle?*

Students in Action

The Grade 4 and 5 students are engaged in a series of geometry activities using geometry tools, folding paper, sharing and discussing ideas and explaining details about their constructed 2-D shapes.

“Students in Action: What do you notice?” is available for viewing without any annotations and is also available for viewing with annotations that identify some of the actions, concepts and relationships.

Five teaching practices for improving the quality of discourse in mathematics classrooms: (1) “talk moves” that engage students/facilitate discussion, (2) the art of questioning, (3) using student thinking to propel discussions, (4) setting up supportive environments and (5) orchestrating the discourse.

Chapin, O'Connor & Anderson, 2009

What do you notice? (without annotations)

(5:24)

Discussion:

- Describe how the students used mathematical tools and materials to learn about 2-D geometry.
- What actions, such as connecting two points to make a line segment, do you notice? How might you annotate these?
- Which concepts, such as a triangle being the space enclosed by three line segments, do you notice? How might you annotate these?
- Which relationships, such as a right triangle being half of a rectangle, do you notice? How might you annotate these?
- Which actions, concepts and relationships might you focus on during a math congress? Why?
- How does what you see within this clip relate to the van Hiele's Levels of Geometric Thinking and Phases of Experiences?

What do you notice? (with some annotations)

(5:24)

Discussion:

- For each item of information shared as text on the screen, what evidence do you notice?
- How else might you annotate this clip?
- What else might you annotate?
- Which annotations relate to the big ideas of geometry?
- Explain how students learn mathematical concepts when building, examining and explaining concrete models.
- How does what you see within this clip relate to the van Hiele's Levels of Geometric Thinking and Phases of Experiences?

Consolidating and Implementing

What did you notice? What did you learn?

(5:56)

The study group summarizes key 2-D geometry concepts and strategies that they learned from the various geometry tasks and problems they solved.

Discussion:

- *Reflect on your mathematical activity and thinking throughout this webcast. Describe key geometric ideas and strategies that you learned.*
- *Which ideas and strategies will you implement in your classroom? Why?*

Ideas for further professional learning include:

- *Implement any of the strategies illustrated during the webcast within your classroom.*
- *Use co-teaching or teacher inquiry/study to continue to investigate elementary geometry.*
- *Replay the webcast in its entirety to revisit ideas and to activate collaborative professional discussion and inquiry.*
- *Select one segment of this webcast to study further.*

Resources and Related Reading

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 36 (5), 412–446.

Chapin, S.H., O'Connor, C., & Anderson, N.C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K–6 (2nd ed.)*. Sausalito, CA: Math Solutions.

Crowley, M.L. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry, K–12*. Reston, VA: National Council of Teachers of Mathematics.

Hunter, S., Kotsopoulos, D., & Whiteley, W. (2007). *Geometry, space and technology: Challenges for teachers and students*. Fredericton, NB: Canadian Mathematics Education Study Group Conference.

Mason, J., Graham, A. & Johnson-Wilder, S. (2005). *Developing thinking in algebra*. London: Sage.

Ontario Ministry of Education. (2005). *The Ontario Curriculum, Grades 1–8: Mathematics (revised)*. Toronto: Queen's Printer for Ontario.

Takahashi, A. & Yoshida, M. (2004), Ideas for establishing lesson study communities. *Teaching Children Mathematics*, (10) 9, 436–443.

Technical Instructions

How to Access the Print and Video Resources

To access the Print and Video Resources folder in Windows, insert the DVD into the DVD drive of your computer and:

1. Click on the Start menu.
2. Select My Computer.
3. Right-click the mouse on the DVD icon titled GEOMETRIC_FIGURES_DVD to open a drop-down options list.
4. From the drop-down list, select and click on the Open option.
5. Double-click on the folder titled Print and Video Resources to access the files. Ignore the folders titled Audio_TS and Video_TS.
6. Select the resources you wish to use directly from this folder, OR Copy onto the Desktop and open files from the Desktop.

Alternatively, when the DVD is inserted and the options box opens:

1. Select the option Open Folder to View Files.
2. Click on the Print and Video Resources folder.
3. Select the files you wish to use directly from this folder, OR Copy the files onto the Desktop and open them from the Desktop.

To access the Print and Video Resources folder in Mac OS X, insert the DVD into the DVD drive of your computer and:

1. Exit from the DVD player (which typically opens automatically when a DVD is inserted in the drive).
2. Double-click on the DVD icon titled GEOMETRIC_FIGURES_DVD
3. Select the files you wish to use directly from this folder, OR Copy the files onto the Desktop and open them from the Desktop.

How to Save the Video Files to Your Computer

The video files can all be copied and saved to your computer using either of the following methods for copying and pasting files.


Method 1

1. Right-click on the file and choose the Copy option.
2. Right-click within any computer folder into which you would like to save the file, and choose the Paste option.

Method 2

1. Left-click the mouse on the file you want to save, so that the file is highlighted.
2. Simultaneously press the Ctrl and C keys (or, for Macintosh users, the Command and C keys) to copy the file.
3. Left-click within any computer folder in which you would like to save the file, and simultaneously press the Ctrl and V keys (or, for Macintosh users, the Command and V keys) to paste the file there.

For Macintosh users, the Command key is the one with the following

symbol: 

NOTE: If you want to insert video files into a PowerPoint presentation, you must save these video files in the same folder that contains your PowerPoint file. If you save a PowerPoint presentation to another location (e.g., a memory stick, CD-ROM, etc.), you must also save the video files in the same location in order for the video to play. So, if you transfer the presentation to another computer, you must also transfer the video files with it, or else the video will not link to the PowerPoint presentation.

How to Insert Video Clips (WMV files) into a PowerPoint Presentation

On this DVD, you will find WMV versions of all segments of the webcast. To insert a clip into a PowerPoint presentation, follow the directions below:

1. Open your PowerPoint program.
2. Create a new PowerPoint presentation OR open an existing PowerPoint presentation, and within it, open the slide on which you would like to add the video.
3. Insert the webcast DVD into the DVD drive of your computer.

4. If a new window opens asking how you would like to view the files on the disk, choose the option Open Folder to View Files; OR

If a new window does not open, open the My Computer window from the Start menu. In the My Computer window, double-click on the icon that is shaped like a disk, which will likely be labelled D: or E:.

5. Save the video segment that you want to insert in a PowerPoint into the same folder that contains your PowerPoint presentation.

NOTE: Video files that have been saved to your computer can be cropped and edited into smaller segments using Movie Maker (free on PCs) or iMovie (free on Macintosh).

6. Open the PowerPoint slide on which you would like to insert the video, and click on the Insert menu in the PowerPoint menu bar.
7. From the Insert menu, select Movies and Sounds, and click on the Movie from File option.
8. A window opens, prompting you to select the video file that you would like to add. Find and select the video file that you saved in step 5.
9. Once you have chosen the video file you need, another window opens and asks whether you want your movie to play either automatically when you enter the slide, or only when it is clicked. Choose your preference. (You will notice that the starting image of your movie is not displayed on the slide.)

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