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Research Monograph #59

How can teachers foster mathematical thinking?

Making Space for Students to Think Mathematically

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Teachers can foster mathematical understanding by …

- providing a safe space for taking mathematical risks, allowing for exploration, and promoting collaborative learning
- presenting rich problems and encouraging high-quality student interaction
- creating a *math talk* learning community
- asking good questions and listening to student thinking

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Contemporary views in mathematics education promote classroom practices that value student voice, develop students' mathematical thinking, and engage students in mathematical inquiry.[1,2] Meaningful mathematics takes place in K to 12 classrooms that support students as they investigate, represent, and connect mathematical ideas through discussion in the context of problem solving.[3,4] Teacher practices that promote inquiry can be challenging to implement, as they cannot be prescribed. Promoting inquiry requires that teachers ask good questions to prompt student thinking. It is equally important that teachers listen and respond to student thinking in order to develop students' mathematical thinking and confidence as mathematicians.[3]

Asking good questions and encouraging students to build on one another's thinking gives students voice and enables them to become more critical thinkers in mathematics.[6] By sharing their thinking, students learn that mathematics is more than a set of rules and that alternative perspectives are possible and valued.[7] Encouraging students to develop their own strategies – rather than simply giving them a set of rules and procedures – fosters both ownership of, and engagement in, the mathematical task.[8] This requires creating an environment in which students feel safe taking risks while exploring alternative strategies that may or may not reach a solution. Establishing socio-mathematical norms, such as ways of phrasing mathematical arguments or making sound justifications, can provide students with ways to discuss and respect one another's ideas.[9]

Implications for Practice: A Case Study

The following description of how "Angela" teaches Grade 8 mathematics provides a window into the practices that foster mathematical well-being.

On mathematical well-being ...

"Angela recalls being frustrated as a student in mathematics class – listening but not hearing a word because she didn't understand. Math, to her, was another language. She carries this experience into her classroom, reminding herself what it is like to feel this sense of isolation."

Create Positive and Safe Classroom Environments

Angela recalls being frustrated as a student in mathematics class – listening but not hearing a word because she didn't understand. Math, to her, was another language. She carries this experience into her classroom, reminding herself what it is like to feel this sense of isolation. As a teacher, she strives to establish a safe and inclusive classroom environment in which students are engaged in mathematics and feel comfortable sharing ideas; she also aims to share her excitement when new ideas or ways of thinking about topics emerge from student contributions. She promotes a *growth mindset* rather than a *fixed mindset* and, thus, encourages students to see themselves as capable mathematics learners.[10]

To establish a similar environment in your own classroom, consider the following:

- The use of resources, such as manipulatives, large paper and markers, document cameras, or other materials, allows students to share their thinking with one another other and with the class and facilitate multiple representations and student interaction.
- The collaborative environment that emerges when student work is visible, and when students and the teacher have space to move around and interact with one another, allows students to engage with different ideas and perspectives and to build on previous learning.
- These complex interactions, when accompanied by meaningful and constructive feedback, show respect for student voice and simultaneously challenge students to think deeply about mathematical ideas.

Provide Rich Mathematical Tasks

Angela often uses a math forum[11] which begins by presenting a rich problem. The example below lends itself to many approaches and strategies and has been used from K to 12:

In the first tug of war, four frogs on one side had a tie with five fairy godmothers on the other side. In the second tug of war, one dragon had a tie with two fairy godmothers and one frog. The third tug of war was between one dragon and three fairy godmothers on one side and four frogs on the other side. Who won this tug of war?

After Angela introduces the task, she observes as students move into pairs to write their ideas, solutions, and strategies. A variety of materials, such as linking cubes and two-colour counters, are available for students to choose from when constructing mathematical models, making conjectures, and connecting their ideas. As they work through the problem, they make connections to previous learning in the class (e.g., proportional reasoning or unit rates) or previous life experiences (e.g., playing Pokémon) or develop new ways of thinking (using substitution in ways that resembled algebraic thinking). Rich mathematical tasks provide varied opportunities for learning and encourage high-quality student interaction.[12]

In choosing rich mathematical tasks, consider the following:

- Rich tasks provide multiple entry points and accommodate a variety of approaches; they can open spaces for students to participate and share their thinking, so that all students can engage with the problem.

- Tasks that have high cognitive demand and multiple ways of solving problems provide opportunities for students' mathematical thinking and discussion and, thus, challenge students to develop their understandings.[13,14]
- Scaffolding students' exploration of a rich task too early can take away students' opportunities to explore and build confidence with solving problems in their own way.
- Affording students the opportunity to explore mathematics without the pressure of imposed steps and procedures or rules can elicit a stronger response, greater engagement, and most importantly, enhanced mathematical understandings.

Create a *Math Talk* Community

As students work on the activity, Angela circulates, validating and building on students' work; she uses questioning to encourage students to further develop their ideas and conceptual thinking. Once students have worked on the problem for about 45 minutes, Angela asks groups to present their solutions to the class. She uses her observations to determine which groups will present, and in what sequence, so that the class can see a range of strategies and can connect different representations of similar strategies. Following each presentation, students are invited to paraphrase what the presenters have shared, to ask questions for clarification, to elaborate on the presentation, and perhaps to challenge the presenters with a possible correction or alternative approach. Angela's approach is similar to what researchers call a *math-talk* learning community. In this community, students are encouraged to ask questions and to explain their thinking. In this way, students become "sources of mathematical ideas"[1] (p. 81).

In establishing a math talk learning community, consider the following:

- Student questioning and explaining are important for developing and consolidating students' thinking and understanding.
- During this process, students are able to compare and connect their ideas and strategies with those of other students.
- Students and their teacher are respected as co-questioners in this environment.

Ask Good Questions and Listen to Student Thinking

The types of questions teachers ask, and the ways they listen and react to student responses, play an important role in developing students' mathematical thinking.[15] Table 1 provides examples of the particular types of questions that Angela used to elicit student thinking.

Table 1. Question Types and Examples that Elicit Student Thinking

Question type	Example from Angela's class
Seeking an alternative method	So you used one strategy here, is there another strategy you could use to represent how you could work on this problem?
Posing new challenges	So, what do you think we should do next?
Promoting group interaction	Okay, do you want to explain to Emily what you are doing?
Encouraging sense-making	Why does it make sense to you?

Interpretive vs evaluative listening …

"When Angela listened to student responses, we saw a few instances of what Davis calls evaluative listening, which seeks to hear a correct answer. However, we saw many more instances of what Davis calls interpretive listening, which seeks to interpret, understand, and make sense of what the student is thinking."

Shifting from evaluative listening to interpretive listening can help a teacher decide how to respond to and support mathematical thinking. Many of Angela's questions provided her with opportunities for interpretive listening: "Can you explain this to me?" "What made you think of unit rates?" "What did you find out when you did that?" We saw her make strong use of these opportunities as she used student responses to these questions to further probe their thinking; each question she asked depended on the student's previous response.

When using questions to elicit and respond to student thinking, consider that:

- Asking questions, listening, and responding to student thinking promotes group interactions and enriches students' mathematical understanding.
- Asking questions that facilitate interpretive listening will prompt student thinking and will provide opportunities to listen to that thinking.
- Asking good questions, such as the types outlined in Table 1, can help to elicit student thinking.

In Sum: What This Looks Like in the Classroom

In Angela's classroom, the questioning, listening, and responding to student thinking promotes a wide variety of types of mathematical thinking. For instance, in response to the sample problem, one student pair used different coloured cubes to represent the frogs, dragons, and fairy godmothers; they determined equalities and made substitutions with the cubes – all aspects of algebraic reasoning. Another pair had a similar strategy of substitution but used symbols rather than cubes. We also saw students using proportional reasoning and notions of equivalency in their use of unit rates and equivalent fractions. Students modelled their mathematical ideas in various ways through the use of manipulatives, words, pictures, symbols, or connections to life experiences.

Our observations confirmed that by sharing these individual strategies students gained a deeper understanding of the problem and their own solutions. As they questioned and paraphrased one another's work, it was apparent that sharing different solutions not only helped students build their repertoire of strategies but also provided them with the opportunity to connect their solutions to other representations and ways to apply the mathematics.

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